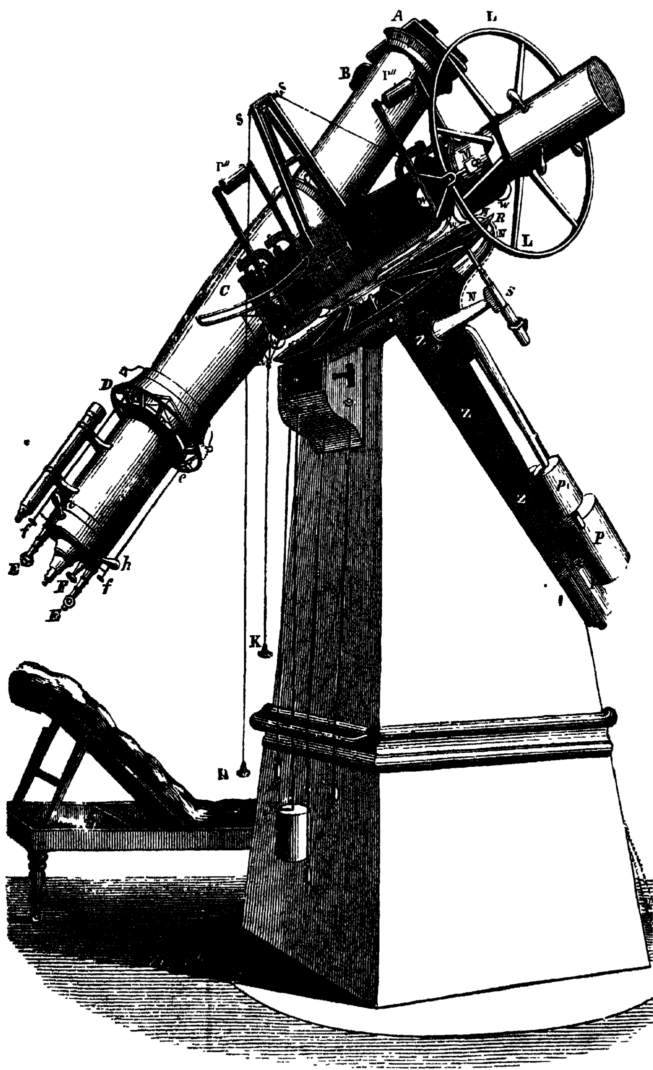
RUDIMENTARY ASTRONOMY.



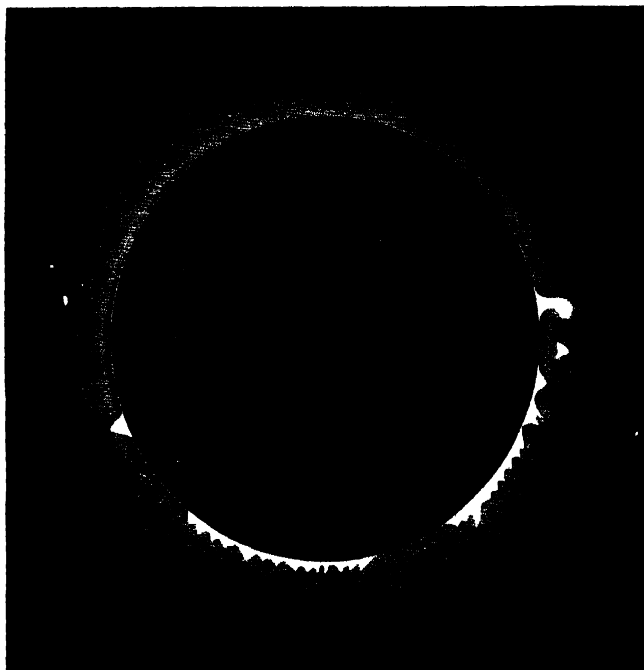
View of the Oxford Heliometer, as given in Captain Smyth's "Ædes Hartwellianæ," or Description of Dr. Lee's Mansion at Hartwell, engraved from a drawing by Mrs. Smyth, and kindly put at the Publisher's disposal.

RUDIMENTARY ASTRONOMY:

BY

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Solar Eclipse of 1851, *July* 28.

LONDON:

JOHN WEALE, 59, HIGH HOLBORN.

1852.

PREFACE.

It might with propriety be asked, what is the need of a new book on Astronomy, when so many excellent treatises already exist in the English language, of every class, both such as are familiar and rudimentary as well as such as exhaust the mathematical theories of the subject.

As far as the publisher of this little book is concerned, it may be sufficient to reply that a treatise was necessary to harmonise with his other "Rudimentary Treatises" on scientific subjects. The author also, when he was requested to write a work on the subject, felt convinced, after some investigation, that there did not at the time exist a book which, in small compass, and in a cheap form, would give the student a sketch of the processes pursued at present in modern observatories, together with the explanations of the leading phenomena of the science, and the most recent results of modern discovery.

There are many catechisms and treatises on Astronomy, some of which form only the introductions to other of the sciences, such as Geography, while others confine themselves to some special branch of the subject. Such treatises are generally descriptive, and confine themselves chiefly to brief

explanations of some of the most striking phenomena, and to the details of the chief facts relating to the solar system and to elementary and sidereal astronomy. More systematic treatises, such as the admirable "Outlines" of Sir John Herschel, are necessarily published at a high price, and are out of the reach of the great mass of the people whom these Rudimentary Treatises are intended to benefit.

Being on these grounds impressed with the conviction that a book which should embrace in small compass the chief *facts* of the science of Astronomy, and which should at the same time familiarise the mind of the student with the *reasonings and processes* by means of which the facts are arrived at, would be likely to prove useful, the author was willing to undertake the task of the compilation, though, from the small space allotted him, he was afraid that it would prove difficult and perhaps embarrassing. He was, however, on proceeding with the work, agreeably surprised to find that much more matter could be compressed into the space than seemed at first to be practicable, by confining himself strictly to the leading features of the science, and by omitting, or passing over with short notice, such casual phenomena as are mere consequences of the general laws that are developed. It is hoped that even in this respect the reader will in general not be disappointed, but that he will meet with an explanation of the greater number of those phenomena which he has been accustomed to find in books more exclusively descriptive; but, if he should find omissions, it is hoped that the expressed object of the author, to deal only with the leading reasonings and facts of the science, will be a sufficient excuse.

The printing of the book had commenced before the author was aware that a treatise by Mr. Hind on the Solar

System, in a cheap form, was nearly ready for publication, and it is gratifying to find that the plan of that work does not interfere materially with the present Rudimentary Treatise. Mr. Hind's object, as explained in his Preface, has been to write a *descriptive* work, and "to present the reader with the latest information on all points connected with the solar system." The author's object, on the contrary, has been to write an *explanatory* work, which should at the same time contain the leading facts of the science, to serve for the purposes of illustration, and to make it acceptable to those who seek only for popular information.

It is hoped that the chapter which has been introduced on Astronomical Instruments and their mode of use, will prove useful both to the general reader and to the student who is preparing to study Astronomy more systematically. The explanation also which it has been found practicable to insert concerning the theory of gravitation, and of some of the leading features of lunar and planetary perturbations, will also, it is hoped, induce the reader to seek for fuller and more philosophical knowledge in "Airy's Gravitation," to which reference has been made more than once, and which, together with the "Ipswich Lectures" by the same eminent astronomer, should be in the hands of every young student who hopes to proceed to the severer reasonings and investigations connected with the mathematical theories of Astronomy.

R. M.

GREENWICH,
March, 1852.

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Page 63, line 5, in the Formula, *for* "L l" *read* "L + l."

RUDIMENTARY ASTRONOMY;

OR,

A CONCISE ACCOUNT OF THE PRINCIPLES OF ASTRONOMY FOR THE USE OF BEGINNERS.

INTRODUCTION.

1. THE science of Astronomy has forced itself earlier upon the attention of mankind, and has been earlier cultivated, than any of the mechanical or physical sciences. This has arisen in part from the grandeur of the phenomena that are forced upon the attention of every one endued with ordinary faculties, and partly from the necessity of *attending* to the most striking of them with reference to the pursuits of daily life. Thus, the hours that can be allotted to daily labour, as well as the vicissitudes of the seasons devoted to various agricultural pursuits, day and night, seed time and harvest, summer and winter, depend upon the apparent motions of the sun, and can only be known or predicted by a study of his motions. As soon, also, as people living on the sea-coast had acquired civilisation enough to know the importance of making the ocean the means of communication between themselves and neighbouring countries, they would recognise the importance of studying the positions and the motions of those points of reference that glittered in the firmament above them, and seemed so exactly adapted to the purpose. They could quickly observe that though, at first sight, the "mazy dance" of the planets and the stars presented inextricable confusion, yet a trifling amount of observation would reduce their motions to something like order.

2. The Chaldean or the Phœnician astronomer, studying

the aspect of the heavens for a whole night, would explain to himself with tolerable precision the nature of the motions exhibited. Looking towards the north at successive hours, he would observe in one part of the heavens that the stars, if not actually at rest, yet had motions very inconsiderable compared with those in other parts, and that they appeared to turn round a point or pole, defined by a tolerably bright star, which itself appeared to be absolutely motionless. Looking towards the east he would observe stars rising successively above the horizon, or earth-bounding line, and equably proceeding in parallel directions towards the south, while in the west those that had previously occupied his attention vanished one by one, and were succeeded by others declining in the same direction. The general direction of the motion of the stars from the east towards the west, and their apparent common motion round some one line or axis, of which a point in the heavens near the north or polar star was the termination, would be quickly recognised.

3. It would also be readily seen that the stars, though thus partaking of a common motion round a fixed axis, had no visible motion with regard to each other. Night after night the same remarkable groups would be exhibited: Orion would appear with his sword and his belt; Castor and Pollux, twins in magnitude, and conspicuous for their brightness, would preserve the same distance from each other and the neighbouring stars; Sirius would glitter with unrivalled brightness beneath them, and the general aspect of the heavens would be unchanged.

4. But there was one fact, which, as the seasons rolled onwards, they would not fail to discover, viz. that the whole of these glittering bodies had a motion with regard to the sun, that is, that either the sun moved amongst them in the course of the year from west to east through the whole circumference of the heavens, or that the stars travelled in one compact mass towards him, from east to west, in addition to their diurnal motion. Beginning their observations soon after sun-set they would observe that, at the same hour of the evening (reckoned by the sun), the stars in the east were higher after a few nights, or had risen earlier, while the stars in the west were lower or nearer to setting, that is, they had all apparently been moving from east to west towards the sun, or the sun had been moving from west to east, which would be by far the simpler and likelier hypothesis. By observing, too, the stars which at

different seasons rose and set very near the sun, they would be able to trace rudely his path among them, or to map out his track in the heavens, that is, they would get a notion of the ecliptic, or the sun's annual circle. By dividing also the globe of the heavens into two equal portions by a plane drawn through the centre at right angles to the axis before mentioned, that is, by the equatorial plane, they would find that the ecliptic cut this plane at two points at the extremities of a diameter, or that the sun's motion was in a great circle of the sphere, and therefore that he moved in a plane which passes through or intersects the earth.

5. Thus far the common-sense observations of the ancient astronomer led him to a knowledge of the general diurnal and annual phenomena of the heavens. But there were other objects which would almost equally attract his attention. The moon, so necessary for the light afforded during the absence of the sun, and so interesting for the variety of the phases exhibited by her, would naturally claim a great share of attention. The track of this luminary could be very distinctly mapped out amongst the stars, and there would be no difficulty in discovering that she too moved in a great circle, that is, in a plane passing through the earth.

6. By following up these observations the ancients would, by degrees, acquire a knowledge of some of the peculiarities of the motions of this body. By tracing her motions, however rudely, they would scarcely fail to discover that the point where her orbit intersected the ecliptic or the sun's path was not stationary, but had a constant retrograde motion, and that in consequence her position in the heavens varied from year to year, and after a time they would gain a knowledge of some of the most remarkable of her inequalities.

7. Amongst the stars, too, one or two conspicuous in brightness would attract their attention by an evident motion of their own. They would see them sometimes move in the order of the signs, or in the direction of the sun, and then, after appearing at rest for a period, the direction of their motion would become changed, and they would move contrary to the order of the signs; and they would not be long before they discovered that the motions of these *planetæ*,* or wandering bodies, were connected with the sun, and they would make it their business to map

* From πλαναω, to wander.

down the motions of these bodies, and to discover the law of them.

8. It is not our purpose, nor have we space, to enter at all in detail on the ancient history of astronomy, and our remarks have been made solely with the idea of pointing out to the student entering upon the science, what is the natural mode of treatment of the subject, to enable the mind to embrace clearly and consecutively its leading features; but this deviation from the plan, roughly sketched, is necessary, that we must in the first place endeavour to obtain some clear notions of the means by which our own planet has been measured and rendered the basis of further operations, before we venture to apply the line and the plummet to those inaccessible bodies that by their distance present so many obstacles to our inquiries.

9. The plan, then, which we propose to pursue is; first, to explain the nature of the proofs of the ordinarily received hypotheses respecting our own globe, including its rotundity, its uniform diurnal rotation round an axis sensibly fixed and permanent, and the nature of the operations by which its actual size and figure have been ascertained.

10. We shall next devote some space to the description of the instruments, and of the mode of using them, by which the heavenly bodies are referred primarily to the centre of this our planet.

11. We shall then treat of certain corrections required to be applied to the places thence deduced, in consequence of the disturbance produced by the sun and moon on account of the earth's spheroidal figure, and in consequence of the progressive motion of light.

12. The motions of the sun, moon, and planets will follow next in order, and we shall endeavour under this head to give some slight idea of the mutual disturbances which they experience in consequence of the law of gravitation, together with the theory of the eclipses originating in the apparent conjunctions and oppositions of the sun and moon.

13. We shall then proceed to discuss those grander features of Astronomy which are involved in the speculations and research of modern astronomers on the numbers and distances of the fixed stars, and in doing this we shall be careful to give in short compass a sketch of the vast progress which has been made during the present age.

14. In the study of any science, in however elementary a manner, some acquaintance with the rudiments of various

other sciences must be presupposed. In Astronomy, an acquaintance with the chief propositions of elementary geometry is indispensable, and the student is under great disadvantages unless he be in some degree acquainted also with plane and spherical trigonometry. We shall assume also that he has read attentively the Treatise on Mechanics in this series, and is consequently acquainted with the nature of force sufficiently to make our remarks on the mutual action of the bodies of the solar system on each other intelligible. It will be our object to divest the subject as much as possible of all unnecessary technicalities, and to explain, as clearly as our very limited space will allow, all the ordinary phenomena.

15. At the same time it must be understood, that the book is only what it pretends to be—rudimentary; and that the student who has tolerably well mastered its contents, should proceed to study the far more philosophical and complete expositions of the subject that are comprised in Airy's Ipswich Lectures, and Sir John Herschel's Outlines of Astronomy.

CHAPTER I.

FIGURE AND DIMENSIONS OF THE EARTH.

16. Our introductory chapter has shown how the most conspicuous of the phenomena of the starry heavens may be investigated and explained by an observer assisted by nothing but ordinarily good faculties, who watches diligently the apparent motions of the heavenly bodies for a whole season. Such a person will be able to account for the *apparent* diurnal motion of the stars from east to west, by a *real* diurnal rotation of the earth on a fixed axis from west to east, and the apparent annual motion of all the stars in a compact mass relatively to the sun by the hypothesis of a contrary relative motion of the sun.

17. All these phenomena will be equally well accounted for, whatever ideas we may entertain of the dimensions and the shape of the earth. But, as it is evident that the earth must ultimately be our basis for measuring everything external to itself, we will begin by inquiring by what means we derive our ideas respecting its size and figure.

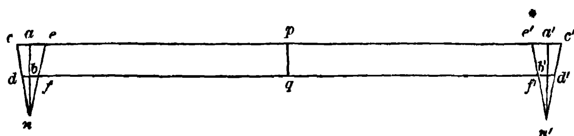
18. First, then, the earth is, roughly speaking, round or spherical, like a ball or an orange. The ordinary proofs of this are of the following nature:—A person standing on the sea-shore, and watching the approach of a ship under sail with a telescope, would first see the topmasts and upper sails, next the mainmast and lower sails, and lastly the hull. Two ships approaching each other under sail, in like manner, first become visible to each other from their respective mast-heads, the lower portions coming successively into sight. Lastly, ships have actually and repeatedly made the circuit of the globe; that is, by sailing out from a certain port in a westerly direction, they have returned to it in an easterly direction, or *vice versâ.*

Lastly, the phenomena with regard to the heavenly bodies, which ought to take place on such a supposition, actually do take place. Thus, to a ship sailing southwards, night after night new constellations towards the south are continually making their appearance, and those towards the north are sinking lower and lower. As the southern cross attracts the delighted attention of the mariner in one direction, his old friends the Greater and the Lesser Bear, with the pole round which they revolve, are vanishing in the other.

19. By such observations we may satisfy ourselves that the earth is, generally speaking, round, but we are still ignorant of its exact shape.* And by what means are we to estimate or measure it? It would seem ridiculous to say that it is principally done by a yard measure or small standard of length, and yet it is really the fact. Small as we are with regard to the vast surface of the earth, and crawling as it were slowly on its surface, it is plain that we cannot put a girdle round it, or measure at once its whole circumference, but, in defect of that, we can measure by degrees tolerably large portions of its surface in various situations, and then by calculation find what form and what dimensions will best satisfy all the observations which have been made. But here a difficulty meets us at the outset. All bodies with which we are acquainted expand by heat and contract by cold, so that, whatever we choose for our measuring rod, it will not preserve an invariable length, and so will be very unfit to measure a distance equal to very many multiples of itself. Various plans have been devised for getting rid of this difficulty, by nicely comparing the metal bars that were to be used with some one standard bar under known circumstances of temperature, or by compensating the bars themselves by means of the application of two metals differently affected by temperature. An ingenious instance of this kind of compensation was first employed in the bars used in the measurement of the Irish Base Line,* along the east side of Lough Foyle, in the county of Londonderry, and afterwards used in India and at the Cape of Good Hope. The principle may be thus explained:— $a\,a'$, and $b\,b'$ are two bars parallel to each other; the upper one of brass, and the lower one of iron, connected by a steel piece $p\,q$; $a\,n$, $a'\,n'$, are flat steel tongues at their extremities,

* See Account of Measurement of the Lough Foyle Base Line in Ireland, By Captain W. Yolland, R.N.; and Airy's Ipswich Lectures, p. 41.

moving freely on conical brass pivots, allowing them to be inclined at small angles with the lines perpendicular to the bars. The lengths *a n*, *a' n'*, are to the lengths *b n*, *b' n'*,



in the proportion of the expansion of brass to that of iron under equal increments of temperature. Then the bars being made of precisely equal lengths at a temperature of 62°, the tongues will for that temperature be perpendicular to the bars, and for any other temperatures the expansions or contractions *a c*, *b d*, *a e*, *b f*, &c. of the brass and iron bars being proportioned to *a n*, *b n*, it is plain by the properties of similar triangles that the points *n* and *n'* will be invariable. The distance between those points, therefore, was used for the invariable lengths of the measuring bars.

20. There was still another difficulty to be surmounted. It was found by experiments that of a brass cylindrical bar and an iron one, with their surfaces equally exposed, the brass heated and cooled considerably faster than the iron bar, and it was necessary to find some means of reducing their heating and cooling powers to the same rate. This was effected by using another principle, viz. that the powers of radiation and absorption of heat depend upon the degree of polish of their surfaces; for, by lacquering the surface of the brass bar, and by browning and lacquering the iron one, it was found quite practicable to equalise the rates of cooling and heating.

21. Having thus obtained an invariable measure of length, it is evidently possible, by using several such bars, to measure a line of any length with perfect accuracy. But other precautions are still necessary in practice. The bars must not be placed close together lest they should disturb each other. It was necessary, therefore, to measure the small intervals between them by means of microscopes, mounted on a similar principle of compensation; other precautions for insuring the bars being in precisely the same level and in the same straight line, it would occupy too much space to detail.

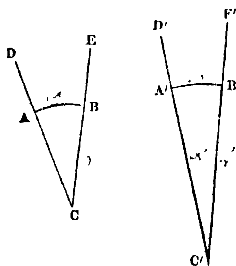
22. However, by this process, suppose we have the power of measuring a line of several miles in length with perfect accuracy. The line must be selected with great care, over a level tract of country, as free as possible from hindrances and obstructions; and the Irish base above-mentioned was admirably adapted to its purpose, running along the sand on the borders of Lough Foyle. Such a line, once measured, is the basis for a large triangulation of the country to be surveyed or measured, and the next instrument required for this purpose is a theodolite. This is an instrument for measuring horizontal angles, consisting, in its simplest state, of a pillar turning freely on a vertical axis, and carrying on outriders, with Y supports attached, a telescope, mounted like a transit instrument, capable of being directed to any object. It has a graduated horizontal circle read by verniors carried by the vertical pillar.

23. Conspicuous objects on the summits of the hills within sight are then selected, and, by means of the theodolite, the angles which the lines joining them and the extremities of the base make with its direction, are then accurately measured. Now, here a curious fact is discovered. When these angles, together with the third angle of the triangle, are accurately measured, it is always found that the sum of the three angles is greater than two right angles; and this circumstance is invariably expected, and made in some degree a test of the accuracy of the work. Now our readers, who are conversant with spherical trigonometry, know that this is always the case in all spherical triangles, the difference being known by the name of the spherical excess, and being a measure of the spherical area. This, then, is an additional proof of the earth's sphericity. Imagine, then, a net-work of such triangles to be measured across a country in the direction of a *meridian*. This word we must, however, define. With regard to the heavens, it may be understood to be that great circle passing through the poles, which bisects the diurnal path of the stars from horizon to horizon, or marks their highest point or point of culmination; or, more technically, it is the great circle passing through the poles and zenith (or point immediately vertical to the observer's position). With regard to the earth, it is the intersection of this plane with the earth's surface, or the circle passing through the poles and the observer's position. Now the object of the chain of triangles, measured in the manner above described, is

ultimately to obtain the length of a determinate arc of the meridian; and the direction of one of the sides with regard to the meridian must therefore be found, or, technically speaking, its *azimuth* must be determined. The term azimuth also requires definition. The azimuth of an object is its direction with regard to the meridian, which we have just defined; and we shall always suppose it to be measured from the south towards the west throughout the whole circle.

24. To determine, then, the azimuth of one of the lines of the measured triangles, we must evidently know the direction of the meridian, and for this purpose the transit instrument is used. As this instrument has not yet been described, we will ask the reader to take for granted, for the present, that by its use a mark can be set up exactly in the direction of the meridian from any one of the stations of triangulation; and then, by the theodolite, the angular distance between this mark and another of the stations can be determined, which will be the required azimuth.

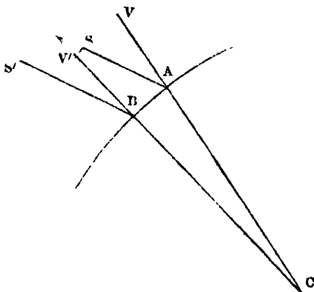
25. With these data, then,—that is, with the calculated lengths of the sides of the triangles and their azimuths,—the length of a portion of the meridian of considerable length, lying between two chosen stations, may be calculated with very minute accuracy. The line thus measured, forming part of the earth's curved surface, will be curved, and the next object is to determine the degree of curvature. To elucidate this, we will explain what is meant by different degrees of curvature. Suppose A B, A′ B′, to be two small equal arcs of different circles, whose centres are C, C′. Draw the radii, and produce them to D E, D′ E′. Then the curvature of these arcs will be measured by the inclination of their radii at the extreme points; that is, by the angles D C E, D′ C′ E′, or, as the arcs are equal, and the angles generally are in the proportion of the arcs divided by the radii, the curvatures will be inversely proportional to the radii.



26. Now mathematicians are accustomed to consider that any curve, so that its curvature be continuous, may at any point, for a very short distance, be supposed to coincide with a circle of a certain determinate radius, called its circle

of curvature, and they may use without sensible error the curvature of this circle to denote the curvature of the curve at this point. To determine, then, the curvature of any parts of the earth's surface, they endeavour to ascertain what is the direction of the vertical line at each extremity of the arc, which they have measured as above explained, and to compare such measures at different parts of the earth's surface.

27. Again, the direction of the vertical line at any place can be determined by three separate methods: viz. in the first place by means of the plumb-line, that is, of a string suspended from a point, with a weight at the end of it; or secondly, by means of a trough of mercury, whose surface when undisturbed is always horizontal, or at right angles to the vertical; or thirdly, by means of a spirit level, that is, of a horizontal glass tube slightly curved, and nearly filled with ether, or other volatile fluid, in which case the bubble or vacant space at its upper surface always retains a horizontal direction. On one or the other of these principles, instruments called zenith sectors have been constructed, by which the zenith distances of stars in the neighbourhood of the zenith can be accurately measured. As all the instruments which we have occasion to advert to in this chapter will be more fully described in the following, we shall beg the reader to take for granted for the present, that the angular distances of stars from the zenith or point immediately over head, can be thus measured, and he will then see very readily how this operation is rendered subservient to the finding of the figure of the earth.



28. Suppose, now, A and B to be two points of the earth's surface, on or nearly on the same meridian; V A C, V′ B C, verticals at the points A and B; S and S′ the apparent positions of the same star, as seen from A and B. Then it is easy to see that, as the lines S A and S′ B must be parallel to each other, the distance of the star being sensibly infinite, the difference of the angular zenith distances, V A S and V′ B S′, is equal to the angle C, formed at the point where the verticals meet each other;

and this angle compared with the length of A B, previously measured, is the measure of the curvature of the earth in the neighbourhood of the points A and B, or, more accurately, for a point at the middle of A B.

29. For the purpose of measuring the curvature of the earth at points widely separated from each other, expeditions have been at various times fitted out by the governments of different countries, but chiefly by those of France, Russia, Prussia, and England. The first great expedition was fitted out by the French government in the last century, and the stations chosen were one in Lapland, as near the pole as was practicable, and the other in Peru, as near as possible to the equator. The astronomers employed were Maupertuis, La Condamine, and Bouguer. The lengths of arc measured were respectively about fifty miles, and two hundred miles, and the general result arrived at was, that for the two stations the inclination of the plumb-line had changed by 1° at the extremities of arcs of 367,086 and 363,626 British standard feet respectively. By the explanation which has preceded, it will be therefore readily understood that the earth was flatter at the northern than at the equatorial station, or the curvature is less near the pole than at the equator.

30. Since that time various arcs have been measured in the great countries of Europe, Asia, Africa, and America. At the Cape of Good Hope, a measure originally made by Lacaille has been recently repeated and verified by Mr. Maclear, the present English astronomer at the Cape. A survey of Pennsylvania was made by order of the British government, in the time of Maskelyne, by Mason and Dixon, the former well known for his Lunar Tables. The Indian government undertook the measurement and triangulation of the immense peninsula under their command; and this great work, begun by Colonel Lambton, has been completed by his successor, Colonel Everest. The triangulation of Russia was accomplished by Struve, as that of Prussia was by his worthy and famous rival, the lamented Bessel. The English survey has been admirably executed by Roy, Kater, and Colby. Denmark was surveyed by Schumacher, whose loss the scientific world has had recently to deplore, and the kingdom of Hanover by the accomplished philosopher Gauss. A large arc, extending from Formentara to Dunkirk, was measured by order of the French Republic, during the first revolution, by the astronomers Delambre and Mechain;

and finally, an arc in the Roman States was measured by Boscovich.

31. The above are nearly all the important measures of the earth's size and shape which have been made, and they all lead to one and the same conclusion—that the general shape is that of an oblate spheroid, flattened at the poles; that is, such a figure as would be produced if a hoop were slightly flattened by pressure, and then made to revolve about the shortest diameter thus produced.

32. We can, of course, give in a short popular treatise no intelligible account of the refined mathematical processes by which the most probable values of the flattening and of the absolute dimensions have been obtained from the measures. It is sufficient to say in this place that the measures have been most elaborately discussed by two of the most accomplished mathematicians of this age, viz. Airy and Bessel, and we will state the results to which they have separately arrived.

33. Airy's results, as given in the *Encyclopædia Metropolitana*, (article "Figure of the Earth,") are,

Equatorial diameter	=	7925·648	miles.
Polar diameter	=	7899·170	

Bessel's results, obtained from the investigation in the *Astronomische Nachrichten*, Nos. 333 to 336, and No. 438, are,

Equatorial diameter	=	7925·604	miles.
Polar diameter		7899·114	

And from both these results it follows that the polar diameter is shorter than the equatorial by about $\frac{1}{300}$ part. This quantity is technically called the *compression*.

CHAPTER II.

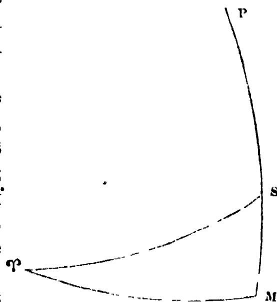
OF ASTRONOMICAL INSTRUMENTS, AND THE MODE AND OBJECT OF THEIR USE.

34. Having in the preceding chapters endeavoured to familiarise the student with the leading phenomena of the heavens, and with the mode of proof by which it is shown that many of these phenomena arise from the equable rotation of the earth round an axis sensibly fixed and permanent; and having finally given some idea of the operations by which the surface of our planet is measured, and its exact size and figure ascertained; we proceed, according to the plan laid down, to give a brief description of the principal instruments which are made use of in modern observatories for fixing geometrically the positions of the heavenly bodies.

35. It is assumed that the student is sufficiently acquainted with elementary geometry to know that the position of a body on a plane surface is defined by means of its distances from two fixed lines (generally at right angles to each other) in that plane; and similarly the position of a body on a spherical surface may be defined by means of its angular distances from two great circles of the sphere, at right angles to each other. One of the circles chosen by astronomers for this purpose is the equator, or the plane passing through the earth's centre at right angles to its axis of revolution, and the other is the plane passing through the earth's axis and the point where the equator intersects the ecliptic or the plane of the sun's apparent motion. This latter point is technically called the first point of Aries; and this designation was given because, in the time of the ancient astronomers, it was situated in the constellation Aries, though it has now retrograded considerably behind that constellation. The angular distance,

then, of any heavenly body, measured along the equator, from the first point of Aries, is called its *right ascension;* and its angular distance north or south of the equator is called its *north or south declination;* or, which is a much better mode of measuring, its angular distance from the pole is called its *polar distance.* In northern latitudes like our own, this distance is measured, of course, from the North Pole, and is called *North Polar distance.*

36. Let, for example, S be a star or other heavenly body, and let the circles ♈ S and ♈ M, as in the figure, be projections of the equator and ecliptic in the sphere of the heavens; P the North Pole, or the point where the earth's axis produced would meet the sphere; and, finally, P M, the projection of a plane passing through the axis and the body S, and therefore necessarily at right angles to the equator. Then ♈ M is the right ascension of the body; S M is its declination (in this instance north); and P S its polar distance.



37. It is the main object of astronomers to determine for all stars in the heavens, as far as is practicable, and for every planet or comet, the values of these quantities. In the former case, that is, for stars popularly said to be *fixed,* we shall see hereafter that their places in the heavens can, by means of certain corrections applied to their observed places, be absolutely determined with a wonderful accuracy; and with regard to the latter, their orbits can be accurately determined, and their places predicted for any time whatever.

38. We will first show how right ascensions are determined. The reader will bear in mind that the stars appear to revolve uniformly round a fixed axis in a certain space of time (called from this circumstance a sidereal day); and for the present we shall assume that they do this with perfect accuracy, and without any disturbance, either by want of absolute parallelism of the earth's axis, or by means of any motion of their own. Imagine then a clock to be set up, and regulated so that its index shall describe very nearly twenty-four hours in the time of a star's passing by the diurnal rotation from

any point in the heavens till its return to it again. It is evident that for the nearer adjustment of the clock and for the obtaining of a knowledge of its rate of going from day to day, it would be necessary to set up a fixed mark to observe this star. For instance, an observation might be made of the time of disappearance of the star, the observer looking along the side of a house whose front is nearly south, and, by watching it from night to night, and taking the time by the clock, an idea might be gained of its approximate rate of going. But, if instead of looking along the side of a house, the observer were to direct a telescope to the star, and watch the time of transit of its image over a fixed mark or wire placed in the plane of the focus of its object-glass, a much more delicate observation would be made; and the observation would be still more accurate and refined if, instead of one wire, several were inserted, and the mean of the times were taken to represent the time of passage. Now this is, in fact, what is done by the *Transit Instrument*, which we shall proceed to describe.

39. This instrument consists chiefly of an astronomical telescope, furnished with a frame of wires at the place where an image of a celestial object is formed by its object-glass, and with an eye-piece, through which the image of the object and of the wires can be distinctly viewed. It is also furnished with a cross axis, passing through its centre, and terminating in two well-turned and polished cylindrical steel pivots, whose axes are, as nearly as the artist can make them, in the same straight line, and at right angles to the optical axis of the telescope. The instrument is placed in bearings fixed to very solid stone piers, and technically called Ys, from their likeness to the letter Y. The position of the piers is so chosen, and they are so constructed, that when the ends of the pivots rest in the Ys, the axis of the pivots shall be very nearly horizontal, and that the optical axis of the telescope shall, in its revolution, very nearly sweep out the plane of the meridian, or the great circle passing through the zenith and the pole. If these conditions were all strictly fulfilled; that is, if the axis of revolution were strictly at right angles to the optical axis of the telescope, and were strictly horizontal; if the optical axis passed through the central wire of the system placed in the focus, and finally passed through any one point of the meridian; then, supposing the pivots to be perfectly cylindrical, and the tube of the telescope perfectly rigid, the

time of a body being on the meridian would be accurately that at which it passed the central wire. But, unfortunately, none of these conditions can ever be accurately fulfilled; and even if they were fulfilled one day, they would not be so on another, on account of the shifting or yielding of the piers, or some part of the instrument. Indeed the great principle of modern practical Astronomy is not so much to get rid of all instrumental errors as, in the first place, to provide means for accurately measuring them, and, in the second, to allow for them when measured in the calculation of the observation. We will then proceed to show how this is managed for the instrument under consideration. It must be remembered that the conditions to be satisfied for a star to be accurately observed as it passes the meridian are three. First, the axis of revolution must be accurately at right angles to the optical axis of the telescope, and the optical axis must pass through the wire or the imaginary line corresponding to the mean of wires over which the star is to be observed. Secondly, the axis of revolution must be horizontal. Thirdly, the optical axis, which (if these conditions be fulfilled) will describe a great circle passing through the zenith, must also pass through the pole. Hence three *errors* are introduced, called respectively the *error of collimation*, the *error of level*, and the *error of azimuth*.

40. To measure the error of collimation, a distant mark is frequently used, set up on a hill at a considerable distance from the observatory, in the direction of the meridian; but a better means is provided in the use of another telescope, set up on Ys in the northern opening of the shutters, and furnished with a wire in its principal focus. By taking off the eye-piece of this latter telescope, and turning its object-glass towards the object-glass of the transit telescope, the wire can be seen through the eye-piece of the latter, and answers the purpose of a fixed mark at an infinite distance. Imagine, then, this mark to coincide with the central wire of the transit instrument, and imagine the eastern end of the axis to be called A, and the western end B. Then, if the instrument be taken out of its Ys, and replaced with the ends of the axis reversed,—that is, so that B be east and A west,—if the wire is still coincident with the mark, there is no error of collimation; but if they do not coincide, the distance between the two will be in angular space the double of the error to be measured. The space in question is measured by means of an apparatus

called a "micrometer,"* with which all good transit instruments are furnished, and which we shall describe presently.

41. Let us now see how to estimate the effect of this error, or to calculate what correction is due to the time of transit of a star from this cause. Suppose the optical axis of the instrument to deviate from the meridian a few seconds (n) towards the east; it will evidently deviate by this same space in any position of the telescope, and the error in time will be that taken by the star in describing this space by its diurnal motion. Now, stars move more slowly as they are nearer the pole, as is very evident from the fact of their describing a smaller circle in the same space of time, that is, in a day. Any of our readers who know anything of spherical trigonometry would easily make out that this slowness of the star's motion increases in the proportion of the secant of its declination or distance from the equator. Hence, since at the equator the error in time would amount to

$\frac{24}{360} \times n^s$ or to $\frac{n^s}{15}$; in any declination δ, the error will be,

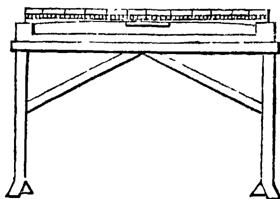
$$\frac{n^s}{15} \times \text{Sec.}\,\delta, \text{ or to } \frac{n^s}{15 \times \text{sin. North Polar dist.}}$$

In strictness, there is another error which acts in the same way as that of collimation, viz. that due to the *diurnal aberration;* but this we cannot discuss now, as the subject of aberration has not yet been brought before the reader.

42. The next error is the error of level. This is measured by a spirit-level, which has been before mentioned, and which we will now describe. It consists chiefly of a hollow glass tube, nearly filled with a fluid of great mobility, such as spirit of wine or sulphuric ether, the unfilled part leaving merely a bubble occupying a considerable portion of the length of the tube. This tube is purposely curved in a slight degree, with its convexity upwards, so that, when the tube is very nearly horizontal, the bubble will rest in a definite position in the upper part. This glass tube is set in a frame of brass, and a scale is affixed to the upper part of it. It is also provided at the extremities of its frame

* From μικρος, small, and μετρειν, to measure.

either with hooks by which it can be attached to the axes of the instrument, or with feet terminating in forks, by which it can be made to ride upon them. Now, suppose it is known that, when the level is attached to an instrument whose axis is horizontal, the ends of its bubble (or, more properly, its centre, which can be known from the scale readings for its two ends) occupy a certain position, then, if the axis cease to be horizontal, it is evident that the bubble will occupy a different position,



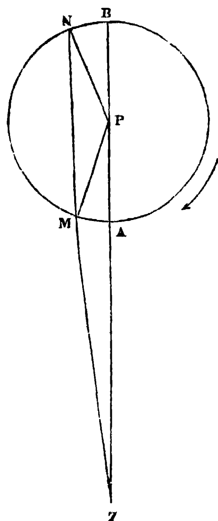
known by reading its scale, and the space through which the centre of the bubble has been moved measures the inclination of the axis. The value of the scale divisions, that is, the error of level corresponding to any number of such divisions, is previously found by some such method as attaching it to a graduated instrument placed with its circle vertical, and finding how many scale divisions correspond to a known space turned through by the circle; or by placing it on a frame called a *level prover*, whose plane can be altered in horizontality by a screw, the inclination of whose thread is known. In the proper use of the level, observations are always made with it in reversed positions, that is, the scale readings are taken for the ends of the bubble, and it is then turned round and placed in the reversed position, and the scale is then read again. We cannot afford space to show how it is applied to ascertain the form and equality of size of the pivots of the instrument; but the student who may wish for farther information may consult some of the works that treat more systematically of astronomical instruments.

43. Imagine, now, that by this means the error of level is found to be m'', the west end of the axis being higher by that quantity; so that for any star the correction will be additive, the telescope being tilted too far east. It is plain that the angular deviation of the optical axis from the meridian will become greater as the object is higher above the horizon, being nothing at the horizon and greatest at the zenith. It will, in fact, vary as the cosine of the zenith distance; and, since the star moves more slowly over this space in approaching the pole in proportion to the reciprocal of the sine of North Polar distance, by reasoning in

the same manner as for the error of collimation, it is plain that the correction to the time of transit will be

$$\frac{m^s \times \text{cos. Zenith dist.}}{15 \text{ sin. North Polar dist.}}$$

44. Our instrument is now, when corrections have been applied for these two errors, reduced to the same state with regard to accuracy as if its optical axis always described a great circle of the heavens passing through the zenith. It will be also observed that the means are purely mechanical by which this has been effected, no reference being made to the heavens in obtaining the errors; but, since the position of the meridian is only known by its being the plane at which the stars generally culminate, or at which they come to their highest or lowest point, it is plain that, in determining the position of the optical axis of the telescope with regard to the meridian, we must have recourse to them. Now, there is a bright star near the pole (Polaris) which everybody knows—the large star, in fact, of the constellation Ursa Minor—which serves admirably for this purpose; and we will show how observations of it are made to determine the deviation of the telescope from the meridian.



45. Let A B N M represent the small circle described by the pole-star on any day round the pole; Z the zenith of the place of observation; and Z A P B a portion of the intersection of the meridian with the sphere of the heavens; Z M N, a portion of the projection of the plane swept out by the central wire of the telescope (the errors of collimation and level being allowed for or got rid of). Draw the arcs P M and P N. Then the star, proceeding in its diurnal course in the direction of the arrow-point in the figure, culminates at A, and is on the meridian below the pole at B, but it is observed by the telescope at M above the pole and at N below. Hence, in culminating, it will pass too

late by the time taken in describing the arc A M, and below the pole it will pass too *early* by the time taken in describing the arc B N. Now, suppose the deviation of the telescope from the meridian to be p'', then the effect of this deviation is nothing at the zenith and has its greatest value at the horizon, and, by reasoning similar to that for the error of level, it is evident that the effect on the time of a star will be in seconds

$$\frac{p^s \times \text{sin. Zenith dist.}}{15 \times \text{sin. North Polar dist.}}$$

Hence, if z and z' be the zenith distances of Polaris above and below the pole, the times of describing the arcs A M and B M will be

$$p \times \frac{\text{sin. } z}{15 \text{ sin. N. P. D.}} \text{ and } p \times \frac{\text{sin. } z''}{15 \text{ sin. N. P. D.}} \text{ or } p\,a \;\&\; p\,b.$$

where a and b are the computed values of the preceding multipliers of p. If, then, t and t' be the observed times of transit of Polaris above and below pole, the times corrected to the meridian will be $t - p\,a$ and $t' + p\,b$, and it is plain that the difference of these times is accurately twelve hours.

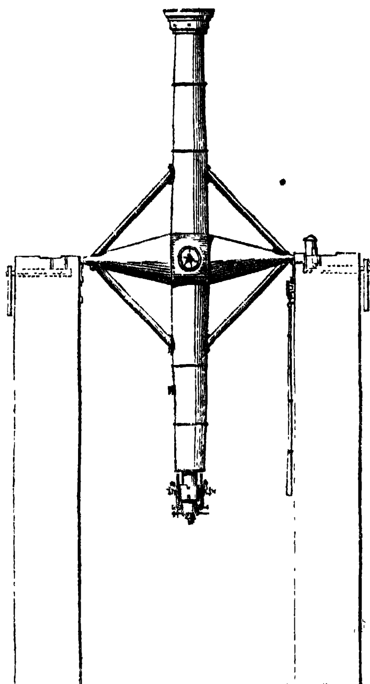
$$\text{Hence,} \qquad t' + p\,b - (t - p\,a) = 12^{\text{h.}}$$

$$\text{or,} \qquad p = \frac{12^{\text{h.}} - t' + t}{a + b}$$

46. If two consecutive transits of Polaris cannot be observed, the error can be obtained by comparing the times of transit of it and some well-known south star, passing the meridian nearly at the same time, with their tabular right ascensions given in the Nautical Almanac, each transit being affected with an error calculable as those above given; but the above method does not require the right ascension of Polaris to be known, though it requires the change of R A in $12^{\text{h.}}$ and the rate of the clock.

47. The following diagram represents a transit instrument mounted on its piers; and its use is, when it has been corrected for the errors above explained, to determine the exact time by the clock at which any celestial object passes the meridian. In its description we have, perhaps, rather

passed the limits prescribed in a popular treatise; but we have thought it above all things desirable to give to persons possessed of a small degree of mathematical knowledge an opportunity of learning, in its simplest form, the nature of this fundamentally important instrument, and the way in which it is cleared of its errors, and made to perform the work which it has to do, according to the practice of the best observatories.



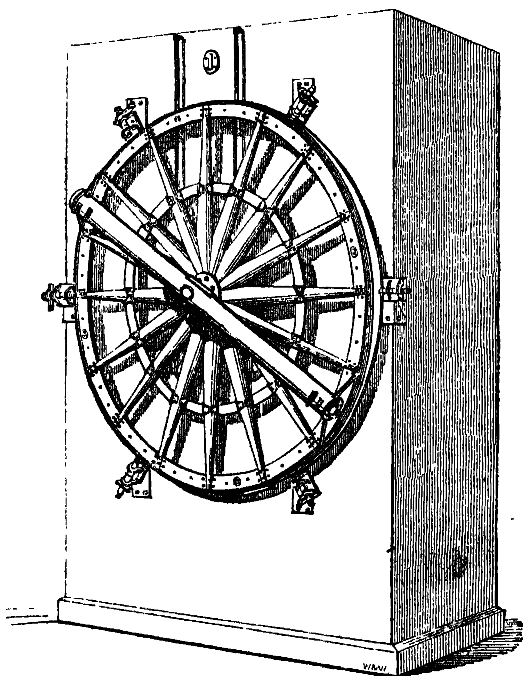
48. The *clock time* at which any star passes the meridian can thus be found with perfect accuracy, and the instrument can then be used for determining the right ascension of all objects whatever, by means of their clock times of transit, in the following manner:—In the Nautical Almanac are given the places of 100 stars, which have been determined by observations kept up unremittingly during the greater ... a century. The tabular right ascensions of several

of these stars observed on any night, are compared with their times of transit over the meridian, and each comparison gives what is technically called a *clock error;* these clock errors again, by comparison on different nights, give the clock rate; and, finally, the clock errors and rates thus found being applied to the observed times of transit of all the objects observed, their right ascensions are found, referred to the same fundamental point as that used in the formation of the "Nautical Almanac" list of stars, and therefore subject to the error of the assumed place of the equinox in that list. It has been previously mentioned that right ascensions are measured from the point of intersection of the ecliptic with the equator, and of course, for a determination of this point, a knowledge of the solar motion is indispensable. Maskelyne's method was to assume provisionally the right ascension of the star α Aquilæ, and with this star to compare the Sun and every other object which he had occasion to observe. He also observed, for some time before and after the vernal and autumnal equinox, the North Polar distances of the Sun; and, knowing the value of the obliquity of the ecliptic (that is, the inclination of the equator to the ecliptic) with sufficient accuracy, he computed the Sun's right ascension from the observed values. These right ascensions compared with the observed right ascensions gave the error which had been committed in the assumed right ascension of α Aquilæ, and afforded means for referring all the objects to the correct place of the equinox. In the present state of astronomy, the same principle is followed, the chief difference being that the assumed right ascensions of several standard stars are employed instead of one.

49. We are thus able to find one of the coordinates necessary for determining the position of a body in the heavens, as referred to the equator; and we must now describe the means used for determining the other coordinate, that is, the North Polar distance.

50. In modern English observatories this is accomplished by the Mural Circle, which instrument we will now proceed to describe. It consists chiefly of a large circle of brass, or other metal, strengthened by several stout spokes or radii, and in general shape resembling a wheel. At its centre it carries a conical axis at right angles to its plane, furnished (next to the circle) with a large, steel cylindrical collar, of about six inches in diameter, intended for the bearing of

the instrument. This axis is inserted into a hollow cone, carried on a plate sunk intò the east or west face of the pier built for the purpose of carrying the circle, and provided with a metal Y support for the steel bearing of the axis, and with means for securing the circle in its proper position for use. A telescope furnished with a conical axis, which enables it to revolve freely if necessary, is securely fastened to the rim of the circle at opposite extremities of a diameter; and it is usual, for the purpose of getting rid of constant sources of error arising from faults in the form or graduations of the circle, to shift occasionally its position on the circle. The telescope carries a frame of several vertical wires in the principal focus (generally five), of

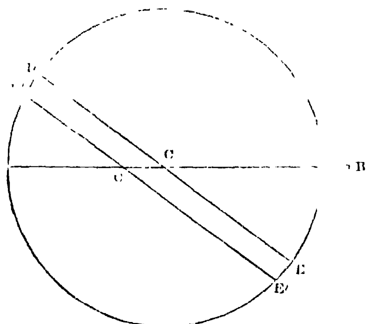


which the middle wire is very nearly in the meridian, and is also furnished with a horizontal wire moveable by a micrometer. It may be proper in this place to explain

that a wire micrometer is an apparatus attached to almost all telescopes used for measuring angular space, consisting of one or more wires stretched across a frame sliding in a box, and carried in a direction at right angles to their length by means of the antagonistic action of a screw and a spiral spring. The screw carries a head, divided generally into sixty or one hundred parts, and an index is fixed beneath this head to some part of the frame or covering of the micrometer for reading the results. When such a micrometer is attached to a telescope, the value in angular space of one revolution of the screw, that is, the angular space through which the wire is carried, by turning the screw once round, may be found by measuring with it the distance between two known stars, or, if the telescope be fixed to a mural circle, by laying the wire by means of the screw of the micrometer upon a distant object, and then finding how much the circle is turned round in bringing the wire again upon the same object after the screw has been turned through a known number of revolutions.

51. Into the cylindrical rim of the circle, at right angles to its plane, is inserted a band of silver or platinum, and the accurate division of this band is the severest test of the skill and care of the artist. It is usually divided into spaces of five minutes of arc, and the whole degrees are marked round the circle, and the spaces of 15′, 30′, and 45′ carefully distinguished, to ensure facility and accuracy of reading. Opposite to this divided band, and fixed firmly to the stone pier, are placed at sensibly equal distances six microscopes, furnished with wire micrometers of the construction described above, the heads being generally divided into sixty equal parts, and so adjusted that five turns of the screw carry the wire-cross very nearly from one division of the circle to another (that is through five minutes of angular space) and therefore that one division on the micrometer-head corresponds to a second of space. The reader will observe that there are three pairs of reading microscopes situated at extremities of diameters of the circle 60° apart. Now, theoretically, if the circle were of perfect form, and perfectly centered and graduated, one reading microscope would be sufficient, but, from necessary imperfections of workmanship, these conditions cannot be fulfilled, and the reading of the six microscopes will, in a great measure, get rid of any important errors arising from any one of these causes, if the instrument be moderately well

constructed. We will confine ourselves to briefly explaining the effect of two opposite microscopes in getting rid of the effect of excentricity, or false centering.



52. A and B represent opposite microscopes for reading the circle drawn in the diagram. The centre of the circle is C, but it turns round a false centre C′. Now, imagine the circle to be used for measuring the angular distance between two stars as they pass the meridian, or the difference of their polar distances (which is the primary use of the mural circle). Let A C′ D′ represent the angle to be measured, and draw D C E parallel to D′ C′ E′; then the circle, turning round C′, will revolve through the angle A C′ D′ for the observation of the second star, and the division at D′ will be under the microscope A, and the division E′ under the microscope B; but, if the circle had revolved round its true centre C, the divisions at D and E would have been under the microscopes. Hence, microscope A would measure the angle in defect by the arc D D′, and microscope B would measure it in excess by E E′, equal to D D′. If, therefore, the means of the readings of A and B be used instead of the single readings of either, the resulting angle will be correctly measured, notwithstanding the excentricity.

53. Mr. Pond, the preceding Astronomer Royal, made many of his first observations by the use of two microscopes; but it is evident that by using additional pairs, distributed round the circle, much greater security is given for the general accuracy of the results, and errors of faulty division and imperfect form of the circle will be in a great degree got rid of; in the ordinary use of the circle, therefore, the six microscopes are always read for every observation.

54. We have said that the mural circle measures only differences of angular space, that is, we have a reading of the circle for one object and a reading for another object, and the difference of readings is accurately the difference of their polar distances; but, if we had an object exactly in the

pole or exactly in the zenith, the differences would be then polar distances or zenith distances.

55. Now, theoretically, a polar point could be obtained (that is, the reading of the circle for an object at the pole) by observing a circumpolar star at its upper and lower passages across the meridian, and then, after properly correcting the results for refraction, taking the mean of the results; but this method would prove troublesome in practice, from the difficulty of obtaining a sufficient number of observations of the same star above and below the pole, in addition to the objection arising from the combination of observations made at an interval of time during which some change might have taken place. A much better method, and that usually practised, is to observe the angular distance between a star and its image reflected in a trough of mercury, since half the difference of the two readings gives at once the altitude, and half the sum gives the reading for an object in the horizon, whence it is easy to get the reading for an object in the zenith, or, speaking technically, the "zenith point." By observing several stars in this way, on the same evening, the zenith point can be obtained with all desirable accuracy; and, hence the zenith distances for all other objects are obtained directly from the circle readings. By a skilful observer, the direct and the reflexion observations can be made perfectly well at the same transit of a star, in the following manner:—A list is prepared of the circle readings of a certain number of stars that can be conveniently or accurately observed by reflexion. A few minutes before the time of transit, the circle is turned till the telescope is in the proper direction to receive the reflected rays, and the mercury trough is also properly arranged. The observer then, after reading the six micrometer microscopes, ascends the stage of the circle, and, viewing the reflected image of the star, bisects it by the wire carried by the telescope micrometer, when, by the diurnal motion, it is brought nearly to the central vertical wire. Then, running rapidly down from the stage, he unclamps the circle, and turns it till the direct rays from the star are received into the telescope, when the circle is again clamped, and the star brought upon the wire by the slow motion screw of the clamp. The six microscopes are then read again, as well as the telescope micrometer, when the observation is complete. This excellent mode of using the mural circle was first used by Mr. Airy, when director of

the Observatory of Cambridge. Another method of obtaining the value of the zenith point, now greatly practised, is by the use of what is called Bohnenberger's eye-piece; that is, of an eye-piece with three lenses, furnished with a reflector either of perforated metal or of glass for illuminating the field of view. By this eye-piece, when the telescope is placed vertical with its object-glass downwards, a distinct view is obtained of the micrometer wire, and of its image reflected from a trough of mercury placed beneath the telescope; and, by the micrometer readings corresponding to the coincidences of the two images, together with the readings of the circle microscopes, the *nadir* point—that is, the point opposite to the zenith—is obtained.

56. Though we are rather anticipating, it is necessary to remind the student that, on account of the refraction which the rays from any heavenly body suffer in passing through our atmosphere, the readings of the barometer and thermometer are necessary for the reduction of every observation made with the mural circle, since the amount of this correction depends upon the density and temperature of the air. We shall discuss this more completely in the next chapter. However, by the foregoing explanation the reader will understand that the *apparent* zenith distance of any heavenly body can be accurately observed, and he must take for granted for the present that the *true* zenith distance can be deduced from this by application of the refraction. It then only remains for us to show how from these results of zenith distance the latitude of the place of observation can be determined, and the polar distances of the objects derived.

57. Let E P Z Q O represent a projection of the meridian; P the pole of the heavens; Z the zenith; E Q the equator; and H O the horizon; Z C being at right angles to H O, and P C to E Q. Then it is evident that Z Q or its equal P H is

the latitude of the place of observation. But P H is the altitude of the pole above the horizon. Hence we have this general rule, *the altitude of the pole is equal to the latitude of the place of observation.*

58. Also Z P, the arc included between the zenith and the pole, is the complement of the latitude, or the *co-latitude.*

59. Now let S S′ be the small circle described by Polaris, or other circumpolar star, transiting the meridian at S and S′. Then in the course of the year a considerable number of observations of the zenith distances Z S and Z S′ may be made with the mural circle, and hence it is plain that Z P, the co-latitude of the observatory, which is half the sum of Z S and Z S′, will be determined. When this is once determined the observed meridian zenith distances can be converted into polar distances by its application, since if $s s'$ be the path of any other body, $\text{P}s = \text{Z P} + \text{Z} s$.

60. For a full description of Troughton's Transit Instrument and Mural Circle, which were for many years used with so much success at the Royal Observatory of Greenwich, as well as for a description and engraving of the great transit circle which has superseded them, the author begs to refer to an article on "Observatories," written by him for "Weale's London and its Vicinity Exhibited in 1851." In this article he will also find descriptions of every class of instruments used in the most celebrated English observatories both public and private.

61. The above instruments, viz. the transit instrument and the mural circle, are the only meridian instruments generally used, but we must not omit in this place a passing mention of the altitude and azimuth instrument, since an instrument of this class has been erected at Greenwich by Mr. Airy, which by its solidity and firmness has produced a series of observations of the moon rivalling those made with the meridian instruments in excellence, and supplying observations for a portion of the orbit of the moon, near her conjunctions with the sun, which could be got by no other means.

62. This instrument consists in its simplest form of a frame revolving on a vertical axis, with which is connected a fixed horizontal circle, graduated like the mural circle. The frame carries Y bearings, for the axes of a telescope included between two vertical circles, one of which is also graduated, and the telescope revolves on its pivots like a transit instrument. Each circle is read by micrometer microscopes

(generally four in number, situated at equal distances round the circles), and in the principal focus of the telescope is fixed a frame carrying six vertical and six horizontal wires. Observations of azimuth and of altitude of a body are made separately in reversed positions of the vertical circles, by clamping separately the vertical and horizontal circles, taking the transit of the object as it passes obliquely through the field, over the vertical or the horizontal wires, and reading the microscopes of the horizontal or the vertical circle, accordingly as azimuth or altitude is the element observed. For determining the deviation from verticality of the vertical axis of the instrument, and the deviation from horizontality of the horizontal axis of the vertical circle, the instrument is provided with spirit-levels attached to its revolving frame, respectively parallel to the vertical circle and the horizontal axis. We have not space, nor is it very important for our object, to describe minutely the manipulations and adjustments of this instrument, but for a minute description of that at Greenwich, we must refer to the article above mentioned.

63. Another instrument which we feel it necessary to describe is the equatorial instrument which plays an important part in modern astronomy. There are some objects, such as comets and the recently discovered small planets, for which a sufficient number of observations cannot be obtained on the meridian, but which can be observed for a considerable length of time by an instrument capable of following them to other portions of the heavens, before or after their meridian passage. Such an instrument is the equatorial, which we now proceed to describe. Its general construction is similar to that of the altitude and azimuth circle, with this important difference, that its principal axis of revolution is *parallel to the earth's axis*, instead of being vertical. The pivots at the extremities of the axis have their bearings on stone piers built for them, the upper pivot resting in a Y, and the lower one working in a socket or hemispherical cup of metal. In that class of instruments most frequent in England (such as the Shuckburgh equatorial at Greenwich, made by Ramsden) the telescope is in the plane of the polar axis, and is carried by a strong frame-work of bars of metal, connected with parallel circular plates, which carry the upper and lower pivots. This frame carries the telescope, firmly fixed between two circles (one of which, called the *declination circle,* is graduated, and the other is used for clamping) by

means of a cross axis (called the *declination axis*), terminating in polished cylindrical pivots; and the telescope, together with the declination circle, is thus capable of revolving freely in planes passing through the poles, that is, in meridian planes. A graduated circle, fixed to the pier carrying the bearing of the lower part of the polar axis, serves to measure the hour-angle, that is, the angle made by the meridian plane with that circle of declination on which the body is when observed, and the telescope carries a frame of fixed wires parallel to the declination circle, and one or more wires moved by micrometers at right angles to them, or parallel to the equator. If the hour circle reads nearly $0^h\ 0^m\ 0^{s\prime}$, when the object observed is on the meridian, then the difference between the sidereal time (that is the clock-time corrected for the clock error obtained by comparison with the transit-clock) and the reading of the hour-circle gives the right ascension of the object observed, subject to the index error of the hour-circle which must be obtained by observations of known stars, and the errors arising from the faults of adjustment of the instrument. Hence it is evident that if a star be once in the field of view of the telescope, and the telescope be clamped, it may be kept in the field by simply turning the polar frame round the axis with a velocity equal to the earth's diurnal rotation, for the telescope itself will sweep out a conical surface, which, when produced, will meet the sphere of the heavens in the diurnal circle described by the star. The greater number of good equatorials are now provided with machinery driven by clock-work for giving this motion; and by these means any measures can be made of objects within the field of view, or any examination instituted, in the same manner as if the objects were at rest.

61. It is necessary, for the complete adjustment of the equatorial instrument, first, that its polar axis be parallel to the earth's axis of rotation—that is, that its elevation be equal to the latitude of the place (57), and that it do not deviate in azimuth to the east or west; secondly, that its declination axis be at right angles to the polar axis; and, thirdly, that the line of collimation of the telescope be at right angles to the declination axis. For the first adjustment, the Y carrying the upper pivot is fixed in a plate admitting of two motions, one in the meridian and one at right angles to it; and the amount of correction is found by observations of stars near the meridian, above and below

the pole, and of other stars about six hours before they pass the meridian and six hours after. The observations in or near the meridian determine the error of elevation of the polar axis, and the six-hour observations the error of azimuth: all the observations being made in reversed positions of the declination circle. The error of collimation is found by observing in right ascension an equatorial star in reversed positions of the instrument, that is, by taking the time of transit over the wires and reading the hour-circle (since, for objects on the equator, the effect of a small error of position of the declination axis produces no error in the time of transit); and, finally, the joint effect of error of collimation and of position of declination axis will be found by observing, in reversed positions of the instrument, a star not very far from the pole. The carriers of the pivots of the declination axis are provided with screw-adjustments for rectification of the axis; and the error of collimation must be got rid of by shifting the position of the wire frame, by means of screw-adjustments with which it is provided.

65. The proper use of an equatorial is to observe *differentially* such objects as cannot be observed sufficiently on the meridian, by selecting some star for comparison conveniently situated, and comparing the positions of the two objects; that is, by obtaining the differences of their right ascensions by their observed times of transit over the wires, and their differences of polar distance by observing each object on the micrometer wire. Even when the instrument is thus used, such observations are generally inferior to those made with the meridian instruments, on account of its want of symmetry and firmness; and the Liverpool equatorial, planned by the Astronomer Royal with his usual regard to solidity, is perhaps the only exception at present existing in the world.

66. The last instrument which we shall describe is the *Zenith Sector*, or instrument for measuring small angular distances from the zenith with great accuracy. This consists of a frame revolving on a vertical axis, and carrying a telescope moving in the plane of the frame on an axis perpendicular to the plane. A graduated circular band, whose centre is the centre of motion of the telescope, is attached to the frame, and is read by a micrometer microscope moving with the telescope. For determining the inclination of the axis of rotation of the frame to the vertical, two methods may be adopted: that is, it may be done by means of a plumb-line, or of spirit-levels. The first method is that which was

used in all the old zenith sectors, and in particular with the celebrated sector used by Bradley. The second method is that employed by Mr. Airy, in the sector constructed under his direction for the great English survey. Another ingenious construction has been devised by Mr. Airy, and is applied to the sector actually used at Greenwich at the present time. An object-glass, with a micrometer attached to the frame that carries it, is fixed to a tube placed vertically over a trough of mercury which is distant from the object-glass by half its focal length, and is capable of rapid and easy reversion. The rays from a star, then, which tend to form an image at the actual focus of the object-glass, are reflected from the mercury, and form an image just above the object-glass; this image is viewed though an eye-piece placed properly to receive it. The star is observed in reversed positions on two wires carried by the micrometer, and the inclination of the tube is measured by a spirit-level placed in the direction of the meridian, though the instrument is so constructed that a small error of level produces scarcely any effect on the zenith-distance.

CHAPTER III.

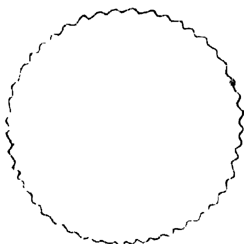
ON REFRACTION, PARALLAX, ABERRATION, PRECESSION, AND NUTATION.

67. If every object in the heavens were really in the position in which it appears to be, and if the points of reference were also fixed and permanent, the business of the astronomer would be comparatively easy; but the fact is the very reverse of this.

68. The objects themselves will appear differently situated to observers on different parts of the earth's surface from two distinct causes, the one being that the rays by which they are seen are bent out of a straight line in their passage through the different strata of the atmosphere, and the other being the spheroidal figure of the earth. The first effect is common to all the heavenly bodies and is called refraction; the second affects sensibly the positions only of those bodies whose distances are not immeasurably greater than the diameter of the earth, such as the Sun, Moon,

Planets, and Comets. The fixed stars are all placed at distances so vast, that the displacement from this cause is absolutely insensible; and even that produced by the motion of the earth in its orbit, which produces for measurement a base of pretty nearly two hundred millions of miles, is also insensible, except in a very few instances, which we shall speak of hereafter.

69. Again, the two points of reference for determining the positions of the heavenly bodies, viz. the pole and the intersection of the ecliptic of the equator, are not fixed. The pole of the heavens, besides having a slow motion, by which it is carried always in the same direction, so as to describe round the pole of the ecliptic a small circle in the sphere in about 25,868 years, is also disturbed irregularly by the action of the sun and moon on the spheroidal earth; so that it does not describe this path quite uniformly, or in an exact circle, but traces out in the heavens a kind of zig-zag line, similar to that given in the figure, and by this means gives occasion for troublesome corrections to be applied to the places of every object observed.



70. Lastly, there is a displacement of all the heavenly bodies, arising from a cause totally different from the former, that is, from the velocity of light combined with the orbital motion of the earth. This displacement was first detected by the illustrious Bradley, by means of a long and excellent series of observed zenith distances of γ Draconis, a star of nearly the third magnitude, passing the meridian of Greenwich within $2'$ of zenith distance, and of which the observations are, therefore, independent of refraction which produces great uncertainties in the observations of all objects whose zenith distances are considerable.

71. Now, before an observation can be rendered available for the ulterior objects of astronomical science, it is necessary that it be corrected for all the causes of displacement which have been mentioned; and therefore, in this stage of our work, it will be necessary to give such explanations of them all, and of the methods of computation by which they are allowed for, as our plan will admit of.

We commence with REFRACTION.

72. Our readers, we will presume, are in some degree

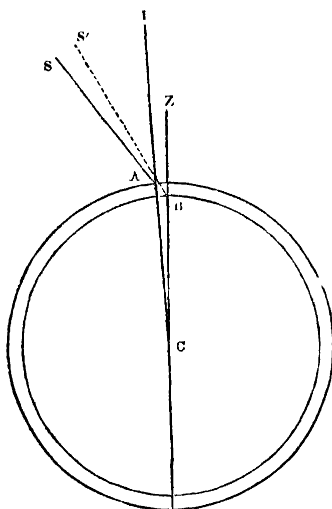
acquainted with the science of optics, or at least they are so far acquainted with it as to be aware that a ray of light, in proceeding out of a rarer into a denser medium,—from air into water, for example,—is bent out of its course in such a way as to make a smaller angle with the perpendicular to the surface separating the two media. The familiar experiment of partially immersing a straight stick in a basin of water will be immediately suggested to the memory of our younger readers: the stick no longer appears straight, but broken at the surface of the water, so that the part below the water appears higher than it really is. The law of optical refraction is this, that for the same media the sine of the angle of incidence is always proportional to the sine of the angle of refraction.

73. Now, the earth is enveloped by an atmosphere extending several miles in height, and of sufficient density to bend out of their course the rays of light proceeding from the stars and planets, which are incident obliquely on its surface. At the zenith the rays fall perpendicularly on the surface, and suffer no refraction; but for every other direction they are bent or refracted, and the more so as they come from a point further from the zenith or nearer to the horizon. Any uncertainty in the amount of this refraction will vitiate an observation of any heavenly body to that amount; and therefore great pains have been taken by various eminent astronomers, not only to discover the laws by which the refraction is regulated, but, by observations made for the purpose, to discover its exact quantity, and, finally, by means of tables, to render the amount in every given case easily calculable. We will proceed, then, to show the nature of the hypotheses on which the formulæ for the construction of these tables are based.

74. If the atmosphere were of uniform density, and if the density were always the same at the same place, the law of sines given above for optical refraction would be immediately applicable; and there would be no difficulty in devising a formula to represent numerically the exact value of the refraction in every case. But the density is, in the first place, not uniform; the strata nearest the earth being the heaviest, and those at the boundary of the atmosphere of extreme tenuity. Again, the density varies according to the temperature, and according to the height of the mercury in the barometer; and the refraction will therefore depend upon these elements, as well as upon the zenith distance.

It is usual, then, to calculate what the refraction will be for some standard readings of the barometer and thermometer; and then, by means of the known laws of aeriform fluids, to calculate how it will be altered, or by what factors it must be multiplied to represent the actual refraction at the time of observation. The standard height of the barometer chosen in English tables is 29·6 in., and the standard temperature 50°; and for any other higher or lower values, the refraction will vary according to the direct proportion of the actual height of the barometer compared with the standard height, diminished or increased in the proportion by which the mercury in the barometer, and the volume of the air, have been increased or diminished by higher or lower temperature.

75. Imagine now a ray of light to proceed from an object whose zenith distance is z; that is, to be incident obliquely on the outer surface of the atmosphere, as is represented in the figure, wherein the inner circle represents the surface of the earth, and the outer circle the boundary of the atmosphere; Z is the zenith, and S A B the path of a ray coming straight from S to A, but bent into a continuous curve between A and B, till it reaches the eye of a spectator at B. The direction in which the object will be seen is determined by drawing the tangent B S′ to the curve at B; and, as the convexity of the curve is turned towards the vertical, it is plain that the apparent place of the object will be above its true place, or the zenith distance is diminished. Now, if the height of the atmosphere were not very small compared with the earth's radius, it would be an extremely difficult problem to determine the whole amount of refraction which the ray has undergone, but as the height is certainly not much above $\frac{1}{100}$th part of the radius of the earth, and the *effective* height, as causing refraction, is much less than this, the supposition of a uniform density is almost sufficiently



exact; and, indeed, the general solution of the problem is in practice subjected to such approximate assumptions, that the result is nearly identical. The result of the investigation is, that, for small zenith distances, the refraction varies as the tangent of the zenith distance; and that, for larger zenith distances, the refraction thus computed requires a correction varying as the cube of the tangent of zenith distance. In very great zenith distances, still nearer approximations are necessary; and below 85° the amount becomes extremely precarious, on account of the unequally heated portions of the earth's surface that the ray meets with in its passage.

76. For obtaining by observation the actual values of the numbers entering into the formula for refraction, circumpolar stars are very usefully employed. γ Draconis, for instance, passes the meridian very near the meridian of Greenwich, and its refraction is scarcely sensible. If, then, the latitude of the Observatory be known, an observation of the mural circle above the pole, compared with one below the pole, will give the whole amount of the refraction for a zenith distance of about 76°; and this observed value equated to the value given in the general formula, will give the value of the constant quantity that enters into it. Bradley employed the greatest and least zenith distances of the sun (that is, the zenith distances at the summer and winter solstices) for obtaining data for the computation of his table of refractions. The best modern tables of refraction are those given by Bessel in the *Tabulæ Regiomontanæ*, and they are exhibited in a much easier form for use in the Appendix to the Greenwich Observations for 1836.

77. In connexion with refraction, we may take the opportunity to make mention of the ordinary phenomenon of twilight. The sun is actually visible to us some minutes after he has really sunk beneath the horizon, by means of the refraction of the rays by which he is seen. But, after he has so far descended beneath the horizon that no rays directly reach us, a portion of his light still reaches us in a secondary way, after being reflected from the vapours of the different strata of the atmosphere and from the minute solid particles of whatever kind which float within it. This reflexion, by which light is brought to us as the sun descends, takes place from still higher strata; the light therefore gets more feeble, and finally ceases, or gives place to total darkness, when the sun is about 18° below the horizon.

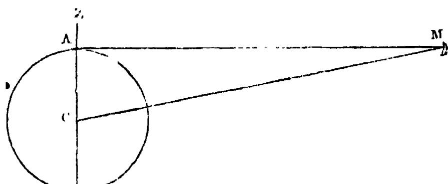
78. The apparent zenith distances, then, which have been observed with the mural circle, are corrected by the addition of the refractions, computed from tables, constructed on the principles above exhibited, and are thus converted into *true zenith distances;* that is, to zenith distances as observed at the surface of the earth. The reader must, however, bear in mind that though the astronomer has thus availed himself of all the resources of science to get rid of the vexatious effects of the atmosphere in disturbing the places of the heavenly bodies, yet, for objects at a considerable zenith distance, an uncertainty yet remains, dependent chiefly on the varying state of local weather and climate, sufficient to prevent the attainment of very minute accuracy in single observations, or even in the means or averages of several observations of the same object. We shall see better the importance of this remark when we come to the more refined speculations of modern astronomy, respecting the distances of the fixed stars and other cosmical problems.

79. In the ordinary processes of astronomy, however, we may assume that the stars are at such distances, as, if not actually infinite, are incapable of measurement. We may also assume, without the risk of the smallest conceivable error, that rays drawn from any one of them to the centre and to any point of the earth's surface, are absolutely coincident; that is, we may assume that the observations have been made at the earth's centre, and referred to a plane parallel to the plane of the sensible horizon of the observer's position. By application, then, of the colatitude, the place of the star is correctly referred to the *true pole*, or the *apparent polar distance* is correctly derived, at whatever part of the earth's surface the observation was made.

80. But, for objects whose distances are finite and measurable, such as the sun, moon, and planets, this is not the case: the polar distances, then, depend upon the observer's position, and before they can be compared, they must be referred to some common point, which will, of course, be the centre of the earth. The nearer the object is, the greater will be the angle between lines drawn from it and the two points of observation on the earth's surface, or the greater the *parallactic* displacement. The consideration of *parallax* follows, therefore, naturally after that of *refraction*, and the laws by which it is calculated for the sun, moon, and planets must be now investigated. It is evident that the most conspicuous effects of parallax will be exhibited in the moon, our

nearest neighbour; and this planet will therefore first claim our notice. Her average distance we shall find in the sequel to be about 60 radii of the earth, or, in rough numbers, nearly 240,000 miles. It is evident, therefore, that when she is in our horizon, the angle made by two lines drawn from her centre, one to the earth's centre, and the other to our position on the surface, will have for its tangent $\frac{1}{60}$, or the *parallactic angle*, or *horizontal parallax*, as it is called, will be rather less than one degree.

81. Thus, if A be an observer's position at Greenwich, C the centre of the earth, and M the moon, the angle A M C is the horizontal parallax; and a certain portion of this, easily calculable, and amounting generally to a quantity varying from 30′ to 50′, according to the zenith distance, is the correction to be applied to deduce the observation from the surface to the centre of the earth; that is, to deduce the *Geocentric Zenith Distance* from the *True Zenith Distance*.

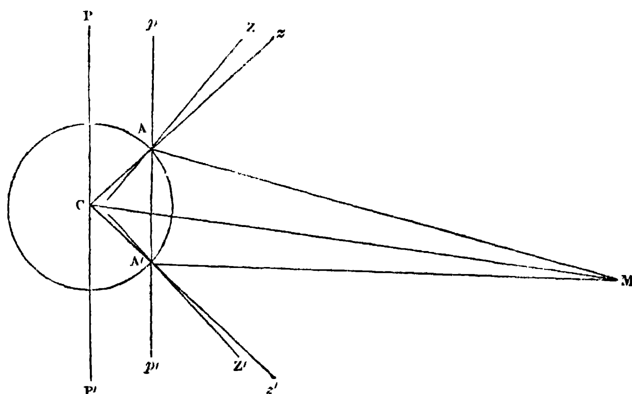


82. We will now see how the amount of the lunar parallax is detected and measured. In the first place, it is evident that the farther separated two stations are with regard to latitude, the greater will be the apparent displacement of the moon's place with regard to the pole, or the greater the observed differences of polar distances. If, then, we have observations made in two observatories, the one situated in a high northern, and the other in a low southern latitude, those observations will be suitable for the purpose of measuring the parallax. It is also desirable that the observatories be situated nearly on the same meridian, since the moon, being a very quickly moving body, will have changed her polar distance considerably in passing from one meridian to another widely separated; and the reduction necessary on account of her orbital motion will not only be troublesome, but will be precarious also on account of the refined knowledge of her motions which it will pre-suppose.

83. Now such observatories exist, the one at Greenwich, in north latitude 51° 28′ 38″, and the other at the Cape of Good Hope, in south latitude 33° 56′, and east longitude $1^h\ 13^m\ 55^s$. The change of moon's polar distance during the

time of her passage from the meridian of the Cape to that of Greenwich, forming so small a part of her whole daily motion in polar distance, can be calculated with sufficient accuracy from very approximate elements of her orbit corrected by her observed daily motion; and we may consider the comparative observations to be made at the same moment of absolute time on the same meridian—of Greenwich, for example.

84. We will now proceed to show how such observations determine the amount of lunar parallax, and consequently the moon's distance on which it depends.



Let, then, A and A′ be two stations on the same meridian of the earth's supposed spheroidal, whose centre is C; M the moon. Draw the verticals A A′ Z′, to the surfaces at A and A′, and A p, A′ p', parallel to the polar axis, P C P′. Join C A, C A′ and produce them to z and z'. Draw also A M, C M, and A′ M. Then the ellipticity of the spheroidal earth being known (see Chapter II.), the angles Z A z, and Z′ A′ z', called the angles of the vertical, which the verticals make with the lines joining the points of observation with the earth's centre, can be computed by the ordinary principles of the Conic Sections, and these being subtracted from the observed zenith distances Z A M, and Z′ A′ M, the angles z A M, and z' A′ M, become known; or, since z A M = A C M + A M C, and z' A′ M = A′ C M + A′ M C, the sum of A M A′ and A C A′ becomes known. But A C A′ is known from our knowledge of the earth's shape, and of the latitudes of the two stations.

Hence we have the parallactic angle at M, or the sum of the parallaxes observed at A and A′. To deduce from this an expression for the lunar parallax, which shall be generally useful in correcting the individual observations, a little more consideration of the law which it follows will be necessary. Confining our attention to the Greenwich station, the parallax is represented by the angle A M C. Now the sine of Z A M (= distance from Geocentric Zenith) : sine of A M C : : Moon's distance, C M : earth's radius at Greenwich.

Hence,

$$\text{sin. AMC} = \frac{\text{sin. ZAM} \times \text{Earth's rad. at Greenwich.}}{\text{Moon's distance.}}$$

Now if at the same time another observation were made at the equator when the moon was on the horizon, we should then have

$$\text{sin. of horizontal equatorial parallax} = \frac{\text{Earth's equat}^{l}\text{. radius.}}{\text{Moon's distance.}}$$

Hence,

$$\frac{\text{sin. AMC}}{\text{sin. hor. eq. par.}} = \frac{\text{Earth's rad. at Greenwich}}{\text{Earth's equatorial rad.}} \times \text{sin. ZAM.}$$

Call then horizontal equatorial parallax = P, and

$$\frac{\text{Earth's radius at Greenwich}}{\text{Earth's equatorial radius}} = r \text{ a known quantity.}$$

Then, Sin. parallax = r sin. P × sin. Z A M.

Let, then, p and p' be the parallaxes, which together make up the whole angle A M A′; and z and z', the corresponding reduced Zenith distances; then,

sin. $p = r$ sin. P × sin. z, and

sin. $p' = r'$ sin. P × sin. z' (r' being analogous to r, for the station at the Cape) also $p + p'$ = A M A′ is a known quantity.

From the above equations the moon's horizontal parallax for the distance C M can be calculated. Hence the distance C M itself can be calculated in terms of the earth's radius, and the shape of the orbit of the moon being supposed known, we can obtain her mean distance.

85. For the sake of such of our readers as are not conversant with mathematical symbols, we may observe, that, in the quadrilateral figure, M A C A′, the sides A C and A′ C, as well as the angles at A, C, and A′ are known; and hence it is plain, by simple geometrical considerations, that the sides A M, C M, and A′ M, can be found. Thus if A A′ be joined,

the angles C A A′, and C A′ A, as well as the chord A A′, can be computed. Hence the angles M A A′ and M A′ A, as well as the side A A′, in the triangle A M A′ are known, and the sides A M and A′ M can be computed. Finally, C M can be computed, or the moon's distance from the centre of the earth.

86. Our readers will now better appreciate the importance of those geodetical operations described in a preceding chapter, by which the earth's size and form have been accurately obtained in terms of an invariable measure of length. The base which has been used in this first instance of the extension of the measure to the heavenly bodies is, in fact, the chord A A′, with the angles at its extremities. The base is small compared with the distance to be measured, and the triangle is not what would be called in Geodesy, a *well-conditioned triangle*. Still it is the best the circumstances admit of, and far better than we can get for the sun and planets, which are at immensely greater distances from us than the moon, and for which, therefore, the base for measurement is comparatively very much smaller.

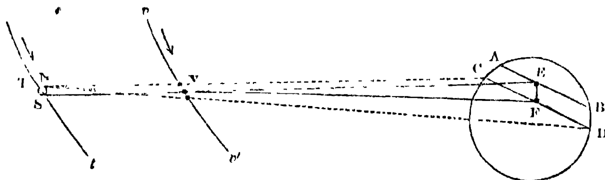
87. Even for the moon a slight inspection of the preceding figure will show that very small errors in the angles M A A′, M A′ A, and therefore in the observed zenith distances, will entail a large error in the distance C M; but if, as in the case of the sun, the base A A′ were, compared with the distance to be measured by means of the angles at its extremities, only $\frac{1}{100}$ part, it is plain that the unavoidable errors of observation and calculated refraction would bear so large a proportion to the angle A M A′, that the result would be unattainable by this method. Now this is precisely the case with regard to the sun, whose horizontal parallax is rather more than 8″, and his mean distance from the earth greater than 95,000,000 miles.

88. Other methods must therefore be employed in this case; and it fortunately happens that there are two methods of observation, one of which can be frequently applied, and the other but very rarely, which have enabled astronomers to get a very close approximation to the value of the solar parallax. The applicability of both these methods depends upon the fact, that there is a law connecting the distances of the planets from the sun with their times of revolution round that body, which enables us, after having found correctly the parallax of any one of them, to infer all the rest. This law we shall enunciate afterwards; but at present it is sufficient to say that advantage has been taken of it, to

make the observations of the next inferior and next superior planet to the earth, viz. Venus and Mars, subservient to the finding of the solar parallax.

89. Venus revolves in an orbit smaller than the earth, and in her revolutions round the sun sometimes, but very rarely, crosses the path of the earth in a direct line between it and that luminary, so as to appear like a black spot on his surface. Such "transits of Venus," as they are called, occurred in the years 1761 and 1769: and the next will occur about the years 1874 and 1882. Dr. Halley was the first astronomer who distinctly saw and appreciated the importance of the observations of the "transits," in relation to the solar parallax; and, in a paper read before the Royal Society not very long before his death, he begged earnestly that every pains would be taken in equipping expeditions for the purpose of observing those which were soon to occur. For observing the transit of 1761, the British government sent out Dr. Maskelyne to St. Helena, and Mr. Mason to the Cape of Good Hope, to make observations to correspond with those which would be made in Europe, in many parts of which continent it would be visible. The only northern country in which the transit of 1769 could be observed, was part of Lapland; and after much deliberation as to the choice of a southern station, it was determined by the British government to send out an expedition to the islands of the South Pacific Ocean, under Captain Cook. This celebrated navigator chose Otaheite for his station, and there the observations were made successfully.

90. The observations made at the two transits have been fully discussed by the celebrated German astronomer Encke, in a book especially devoted to the purpose. Our object at present will be merely to give a brief sketch of the principles of the methods employed in deducing the parallax of the sun from the observations.



In the above figure, suppose A C D B to represent the disc of the sun at a time near the middle of the transit; V the

place of Venus, and T that of the earth. Draw lines through N and S, the northern and southern stations on the earth, through V, till they meet the sun's disc at E and F. Then the planet will appear to the observers at N and S like a black spot on the sun, at F and E respectively; and the more widely separated the stations are with regard to geographical latitude, the larger will be the interval E and F, and the greater the difference of the chords A B, C D, described by the planet in its transit across the disc. As the student has not yet been introduced to a knowledge of the motions of the planets, nothing but a bare outline of the problem can be rendered intelligible; but enough can be shown to exhibit the principle of the method. The earth and Venus both revolve in the same direction round the sun in planes inclined to each other at a small angle, the velocity of Venus being greater than that of the earth. The times of revolution of the planets are also connected with their mean distances by a very simple law, so that the times being obtained by observation, the proportions of their distances can be inferred. At the time of a transit of Venus, the planet must either in descending beneath the ecliptic, or in ascending above it, meet it in a point which is in a direct line with the earth and the sun; that is, technically speaking, it must be in its *node* when in conjunction with the sun; and this is the condition for determining whether a transit will take place. Let the earth be moving in the direction T *t*, and the planet in the direction *v* V *v'*, and suppose, for simplicity, that the plane passing through the two observing stations N and S and the earth's centre, is perpendicular to the ecliptic, and that the rotation of the earth is neglected; and suppose, finally, that the earth being absolutely at rest, Venus moves in the direction V *v'* with the excess of her velocity over that of the earth, which, in fact, represents her relative motion properly. Then, to a spectator at N, Venus will appear to enter upon the sun's disc at C, and, traversing it in the sensibly straight line C D, to go off at D. Similarly to the observer at S, the planet will appear to describe the more northerly line A B, entering at A, and going off at B. The principal observations at each station will consist in taking accurately the times at which the ingress and egress of the planet take place. On entering upon the disc a small, but very perceptible, notch will be made instantaneously on the limb, which can be

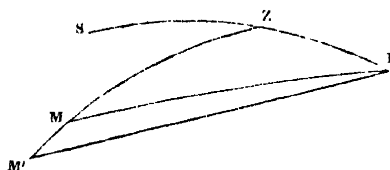
observed with very great accuracy. Then when the planet is completely on the disc, the time of separation of the limb can also be observed, but not with so great accuracy: however, the mean of these times, corrected for curvature of limb, will represent very well the time of ingress of the centre, and similarly the time of egress will be observed. Hence the time taken by the planet in describing the chords A B, and C D, will be known. The velocity, too, of Venus is sufficiently well known from the tables of her motion; hence the angular measures of A B, C D, will be known, and consequently the versed sines of the chords can be calculated. Thence, finally, we have the measure of E F, as seen from the earth. Now the mean distance of Venus from the sun is to that of the earth, nearly, as 72 to 100; and therefore the distances of Venus from the sun and earth respectively, are as 72 to 28 nearly, or as $2\frac{1}{2}$ to 1. Consequently, E F, measured on the disc of the sun, is about $2\frac{1}{2}$ times the arc which the earth's diameter would subtend there at the distance of the earth, or five times the arc which would be subtended by the earth's radius; that is, five times the sun's horizontal parallax. Any errors, then, which are made in the observations, will, as affecting E F, be divided by five, and hence the accuracy of the method is apparent. We have in effect the sun's parallax represented to us on a greatly exaggerated scale, and can take advantage of it to measure this important element of our system with an accuracy attainable by no other method. It will not be difficult to show what will be the effect, on account of the earth's rotation, of choosing situations differing considerably in longitude; but it is rather too complicated for elementary illustration.

91. Besides the British expedition fitted out for observing the transit of 1769, others were equipped by the French, Russian, and other governments, and great pains were taken in choosing such stations as would produce the best data for the solution of the problem. On the whole, the observations were very satisfactory, and the resulting horizontal parallax of the sun was $8''\cdot5776$, which, it is believed, does not differ $\frac{1}{200}$th part from the truth. This gives the mean distance of the Sun about 95,300,000 miles.

92. The last method which we shall mention for finding the solar parallax is by observations of the planet Mars; and, indeed, with the exception of that above described, it is the only method which can give results of moderate accuracy.

Mars is the nearest planet describing an orbit exterior to the earth, and at certain times approaches so near that his distance from the earth is only one-third of that of the Sun, or so that his horizontal parallax is about 27″. Now, this is a quantity which can be measured by comparative observations made at Greenwich and the Cape of Good Hope, with considerable accuracy, by means of methods which get rid of that pernicious influence exercised by refraction on all delicate comparisons of polar distances absolutely measured. Suppose, for instance, the polar distance of Mars be compared, both at Greenwich and the Cape, with that of a neighbouring star, by means of the micrometer attached to a mural circle or an equatorial. The absolute polar distance of the star as observed at both places is the same, so that the differences give immediately the effect of parallax; and these are either altogether independent of refraction or the effect is calculable without sensible error. The remaining errors are chiefly those of observation, and, these being purely casual, can be got rid of by sufficiently increasing their number. The only objection to the method is, that the stars lying in the path of Mars are generally so small and faint, that they are observed with difficulty by the ordinary meridian telescopes; but the great transit-circle has now done away with the objection as regards Greenwich, and if the interest due to the method had not been materially weakened by the far greater accuracy attainable by observations of a transit of Venus, it would devolve on the Government to set the question at rest by the establishment of a similar instrument at the Cape.

93. There is still one way in which the parallax can be deduced by means of extra-meridional observations of Mars made at one and the same station; but we mention it only as belonging to the theory of the subject. Let P Z S be a portion of the meridian, Z the zenith, P the pole, and M the position of the planet Mars, east of the meridian as seen from the earth's centre, but depressed as viewed from the station whose zenith is Z in the vertical circle Z M to M′. Then, the *parallax in time* by which the planet would come



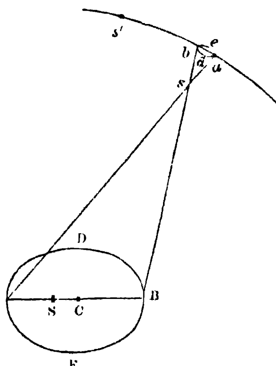
too late upon the wires of an equatorial instrument is measured by the angle M P M′, and by this quantity would its observed right ascension be too great. Similarly, if observed west of the meridian, the observed right ascension would be too small by a similar quantity. If Mars, then, be compared with a star lying in the same parallel of declination (by taking the clock-times of transit of both objects over all the wires) several hours before passing the meridian, and again several hours after, these right ascensions, when reduced by the known motion of the planet to the same time, will differ by a quantity depending on the parallax, and from which it can be calculated. In practice, however, the observations cannot be made with sufficient accuracy to get results available in the present state of astronomy, even if we omit to mention the difficulty of obtaining the requisite observations at sufficient distances on both sides of the meridian.

94. Before leaving the subject, we will briefly mention parallax in its connexion with the fixed stars. We have before said (79) that the distance of the nearest of the stars is so great that it would be absolutely hopeless to endeavour to detect any displacement by observations at different points of the earth's surface; but this might not be the case with observations made at opposite points of the earth's orbit, which presents a base for measurement of about 190,600,000 miles. Surely, we might imagine that, however vast the distances of the stars, unless they were actually infinite, we should be able to detect in them some difference of position by observing them at intervals of six months, during which time the earth has shifted its position by this enormous quantity.

95. And so reasoned Flamsteed, and several succeeding astronomers, in the infancy of accurate theory and accurate observing. Before the discovery of the aberration of light and the nutation of the earth's axis by Bradley, all their efforts were directed to the discovery of annual parallax, and the stars seemed to be removed to greater distances precisely in the proportion in which observations were rendered more accurate by improvements in instruments and the use of them, and as the causes of the displacements, at first rashly attributed to parallax, were accurately known. At length Bradley announced, as the result of his own at that time incomparable observations, that, if a parallax of any star existed to the amount of one second, he should

discover it; and since that time the ingenuity of astronomers has been employed in inventing methods for the estimation of quantities much less than this, by observations and instruments especially devoted to the purpose. We will see in the first place what is the nature of the displacement to be measured.

96. Let A D B E be the orbit described by the earth round the sun, S, in the course of a year; A B, two opposite points of the orbit; *s* a star *not* at an infinite distance (that is, at a distance that may be discoverable by observation). Draw A *s*, B *s*, and produce them. Imagine also the cone which would be swept out by one of these lines during the progress of the earth in its annual circle; then it is evident that the star itself will appear to describe the small orbit, *a d b e*. Now, these points, *a d b e*, lie sensibly in a plane, cutting an oblique cone in a section not parallel to its base, and will therefore be an ellipse described about its mean place. Suppose, now, we had another star, *s*′, at an immeasurably greater distance than *s*, and situated, for example, in the arc *a b* produced; it is plain, by looking at the figure, that its angular distance from *s* would appear to an observer on the earth's surface to vary, being least at *b*, greatest at *a*, and having its mean value at *d* and *e*. If, then, we had an instrument by which we could measure *very accurately* the distances of the stars *s* and *s*′ in angular measure, the variations, when observed, would give us means of determining the size of the apparent orbit described by *s* round its mean place, and from this we could determine the parallax of the star, and consequently its distance from us.



97. Now this is precisely what was done a few years since, by the celebrated astronomer, Bessel, with regard to a remarkable star in the constellation Cygnus (61 Cygni), by means of an equatorially mounted telescope, called a *heliometer*. A heliometer, (so called from the facility with which the sun's diameter and other large arcs can be measured with it)

consists, in fact, of a telescope with its object-glass divided into two equal parts, by a section through its centre. These parts are placed in contiguity in the tube of the telescope, so as to lie in the same plane, like an undivided glass, and, by means of an apparatus of rods and screws, acted on from the eye-end of the telescope, are made to slide past each other, and a scale is provided which measures the degree of the separation of their centres. By this means, each half object-glass produces separate images of the objects observed, and the angular space by which they are separated will be proportional to the distance of the two centres of the half object-glasses. There is also an apparatus provided, by means of which the object-glass can be turned round in the tube, so that the direction of the two images (which is always parallel to the common section of the glasses) can be put into required positions. If, then, they be put into the direction of the diurnal motion of the star, the times of transits of its images, over a wire at right angles to their direction, will, when properly reduced, give their angular distances in terms of the scale divisions; and, the value of the divisions of the scale being thus known, the instrument may be used to measure the angular distances of stars or other objects not far apart. Thus, let A B be two stars, and suppose the screw, acting upon the parts of the object-glass, to be turned, and the object-glass itself to be turned round in the tube, till the separate images occupy the positions A, B; *a*, *b*, *a* being at a very small distance immediately below B; then it is evident that the object-glasses have been separated by a quantity representing the angular distance of A and B. In like manner, the images *a* and *b* can be brought into similar positions, *a*′, *b*′, on the other side of A, B. It is evident, then, that for these different positions of the images, the difference of scale readings corresponds to the double of the distance of A, B; and this can be reduced to arc, by the known value of the scale.

A B
a′ *b*′ *a* *b*

98. Now the star 61 Cygni is double, consisting of two stars of nearly equal magnitude (the 6th), at a distance of a few seconds from each other. The star forms what is called a binary system; that is, it is proved, by observations continued since the year 1781, that the two components revolve round each other, being connected by gravity, or by some similar tie, like our sun and planets.

They also move together through space, with a large proper motion amounting to several seconds per year. From these circumstances, all astronomers have been led to look upon this star as probably near enough to us, to enable us to detect its distance by observation; but it was not till Bessel made his celebrated observations with the heliometer, that any satisfactory result could be obtained. He found two stars very minute, but perfectly observable, each at a distance of several minutes from 61 Cygni, the one lying nearly at right angles to, and the other nearly in the direction of the components, and he made a long series of measures of their distances from the middle point between the components. These measures, when properly treated, exhibit a series of fluctuations agreeing with those which would arise from annual parallax, and leaving no doubt of its existence in the minds of persons familiar with the subject. The result is, that the annual parallax, that is, the angle which the radius of the earth's orbit subtends at the distance of the star, is rather more than three-tenths of a second of space, corresponding to the enormous distance of 600,000 radii of the earth's orbit; or, to a quantity greater than 600,000 times 95,000,000 miles.

99. This, then, one of the nearest of the fixed stars, is placed so far out of the range of the solar system, that, even using our distance from the sun as the unit of measure, imagination almost fails us in conceiving the enormous interval that separates us from it. We are lost in endeavouring to get a fixed idea of the magnitude of the starry heavens, even when using as our basis this measurable distance; and the pride which the consciousness of this wonderful discovery raises in us with regard to man's intellect, has its proper antidote in the inconceivable magnitude of the works of the Almighty, in this our first step to a survey, compared with our petty dwelling-place.

100. There are two other instances in which it is tolerably certain that the amount of parallax has been pretty accurately measured. The first is for a double star called α Centauri, of the first magnitude, in the southern hemisphere, in which all the elements that denote the proximity of 61 Cygni are found existing, in addition to great brightness. The other is the well-known star α Lyræ, of the first magnitude, in our own hemisphere. The latter was detected by the Russian astronomer, Struve, about the same time with Bessel's discovery, and by a similar mode of measurement, excepting

that he used a wire micrometer, and compared α Lyræ with only one star. The former was detected in the first place by meridian observations made at the Cape of Good Hope by the late talented and lamented Professor Henderson, and has since been verified by other observations of the present astronomer at the Cape, Mr. Maclear. The resulting parallax of α Lyræ is about two-tenths of a second, but that of α Centauri amounts to very nearly a second of space. The readers of this little book will see, if they have attentively studied our remarks on refraction and other sources of uncertainty as affecting meridional observations, that the nature of the evidence proving the existence of so large a parallax for α Centauri is inferior to that for the other two stars; and we may still desire, at some future time, a verification of it, by similar differential measures.

101. We will briefly recapitulate a few of our leading remarks on parallax. *Parallax*, generally, is the displacement which a body not infinitely distant suffers by being viewed from the surface, instead of from the centre of the earth. It is measurable in the case of the moon by making observations of polar distances, at stations, one in a high northern, and the other in a high southern latitude, such as Greenwich and the Cape of Good Hope; the base of measurement being the distance of the stations. The results prove that the mean horizontal equatorial parallax (that is, the parallax on the horizon, for a place situated on the earth's equator, at the moon's mean distance) is rather less than 1°; or the moon's mean distance is about 240,000 miles.

102. The parallax of the sun has been found accurately, by means of observations of the transits of Venus that occurred in 1761 and 1769, made at high northern and low southern stations; and the only other method promising any success is that of observations of the planet Mars when nearest to us. The resulting parallax by the transits of Venus is about $8''{\cdot}6$, which corresponds to a mean distance of about 95,000,000 miles. The mean angular diameter of the sun also being about $32'$, this gives for the linear diameter of this stupendous luminary the astonishing quantity of about 880,000 miles. To form some idea of this immense globe, let the student imagine the sun's centre to coincide with that of the earth; then the surface would not only include the lunar orbit, but would extend nearly as far beyond.

103. For calculation of the parallax p to be applied to the zenith distance z of any planet, we have (if r be

the proportion which the earth's radius, at the station of observation, bears to the equatorial radius, and P be the equatorial horizontal parallax),

$$\sin. p = r \sin. P \sin. z.$$

which, in the case of the sun, becomes

$$p = 8''\cdot6 \times r \sin. z.$$

of which a table can easily be formed.

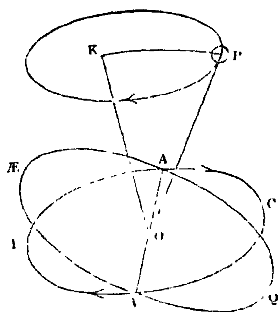
104. Lastly, the distances of the stars can only become known by observations made of them at opposite points of the earth's orbit; and the method which promises and has thus far been attended with most success, is that of measuring their angular distances from other stars probably at a much greater distance. The parallaxes of three stars, 61 Cygni, α Lyræ, and α Centauri, have been found, and amount respectively to about 0″·3, 0″·2, and 1″.

105. Thus far the displacements of objects which we have been considering have respect only to the local position of the observer on the earth's surface, and to the bending of the rays by which they are rendered visible on coming into the earth's atmosphere. By the corrections applied for *refraction* and *parallax*, all the heavenly bodies are referred to the positions in which they would be seen by an observer at the centre of the earth, by means of rays coming to the eye without any deflexion. But it is to be remembered that the positions of all celestial objects have been defined by reference to two points, the one, viz. the intersection of the ecliptic and equator, forming the zero point of right ascension; and the other, viz. the vanishing point of the earth's axis produced, forming the zero of polar distance. If, then, these points were fixed, astronomers would have no farther trouble in settling the positions of all the heavenly bodies but such as arise from a purely optical cause which we shall afterwards explain. But the fact is not so. It has been known as a fact of observation for about 2000 years that the equinox, as it is called, or the point of intersection of the ecliptic and the equator, moves backwards, that is from east to west, along the ecliptic with a motion amounting to about 50″ per year, and that the pole of the equator is thus carried round the pole of the ecliptic, in the same direction; so that it would describe a complete circle about that pole in about 26,000 years. Before, then, we can definitely fix the places of the planets

for definite times, or form catalogues of stars for definite epochs, it is necessary that we should be able to trace the cause and to calculate the amount of the corrections due to this phenomenon. Now this we cannot do without referring to the principles of physical astronomy, and treating, though briefly, of the way in which the revolving protuberant matter at the earth's equator is affected by the sun and moon's action. But it will, perhaps, conduce to clearness of conception of this difficult subject, if we first consider the phenomena of precession and nutation in relation to their effects derivable from observation.

106. We have said that the right ascensions of all stars are measured from the point of the vernal equinox; the position of this point being determined by observing, with the mural circle, the polar distances of the sun a little before and a little after he crosses the equator, both at the vernal and autumnal equinox. The polar distances of the stars are also observed by means of the mural circle. Now, by knowing the inclination of the equator to the ecliptic or plane of the sun's motion (which is, in fact, the sun's distance above or below the equator at the summer and winter solstices, when he attains his highest and lowest points respectively in the heavens), the positions of the stars can be calculated with reference to the ecliptic and the same equinoctial point; that is, their latitudes and longitudes can be calculated. Now it is found that for all stars whatever, observed at distant intervals of time, the latitudes are sensibly constant, but that the longitudes of all have a yearly increase of about 50″. Of course, we may here apply the same reasoning which we have before used with regard to the apparent diurnal motion of all the stars from east to west, and conclude that this common motion of all in the same direction is not real, but apparent, and arises from a uniform *regression* along the ecliptic of the point of reference, that is, of the equinoctial point. This *regression* is called the *precession* of the equinoxes, because its motion being in the direction of the diurnal motion, it brings the intersection of the ecliptic and equator on any meridian sooner each succeeding year than it would have otherwise come, and makes the interval between a season of one year and the corresponding season of the next shorter than it would otherwise be. But the student must remember that the point in question goes *backwards*, and not *forwards*, upon the ecliptic. In the accompanying figure, let A E V O and A Æ V Q represent the ecliptic and equator, and K and P

their respective north poles (O K and O P being at right angles to them), V the vernal and A the autumnal equinox; then the points V and P regress along the ecliptic, in the direction of the arrow points, and, of course, carry the point P in a small circle round K in the direction also indicated by the arrow point. The pole of the equator, in its progress round that of the ecliptic, will move away from certain stars which it has passed in its course, and approach others. Thus, at present it is still approaching the remarkable star α Ursæ Minoris (Polaris), but after a time it will recede from it, and after the lapse of a great many ages, it will approach very near to the great star in the constellation Lyra, which will then be the polar star to astronomers in the northern hemisphere.



107. But though we have described the effect of the precessional motion by means of the small circle, which, in its mean effect, it causes the pole of the equator to describe round the ecliptic, yet this is true only approximately. If we were to lay down the curve actually described, it would be an irregular line approaching to a circle, and with its irregularities recurring again and again, at intervals of about nineteen years. In fact, the true state of the case may be accurately enough represented by imagining a point P to be carried uniformly round, while another point (the real pole) is describing round this a minute ellipse, whose longer axis, about 18½″ is directed towards K, and the shorter axis, about 14″, is at right angles to it. These variations of the mean precessional effect are called *nutation;* and, of course, affect the places of all bodies referred to the pole and to the equinox, and must rigorously be taken into account in all calculations for reducing them to any fixed epoch.

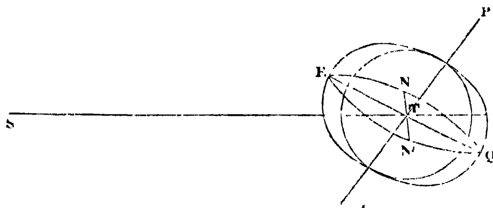
108. We will now endeavour to give some idea of the physical cause of this most interesting phenomenon, whose observed effect was first detected by Bradley, and given nearly in the shape in which we have put it above.

109. Our readers have, doubtless, some notions of the theory of gravitation. If they have not, it is better to omit for the present any attempt at understanding what follows. The law of universal gravitation is simply this, that every particle

of matter in the universe gravitates towards every other particle, or, attracts and is attracted by every other particle, with a force varying inversely as the square of the distance. This law, simple as it is, will account for all the wonderful phenomena which are the objects of our speculation and research in astronomy. By it we can account for the spheroidal figure of the earth and planets, and for the forms of their orbits; and by it we can calculate with wonderful accuracy the place which any one will occupy in the heavens at any assigned period. By the application of this law we can trace out in the heavens the path of so filmy and vaporous a body as a comet, through whose densest parts the smallest stars are visible; and astronomers have, in many instances, predicted their return, after the description of orbits of almost inconceivable magnitude. The tides of the ocean also rise and fall in accordance with the law, and, independently of the local circumstances which affect them at individual places, afford one of its most beautiful exemplifications. Lastly, the present phenomena of precession and nutation admit of a full and perfect explanation, though we can promise to give, in a popular form, only a very vague notion of them.

110. The earth, as you are aware, is a spheroid of revolution revolving round its smaller axis, whose ellipticity, that is, the ratio of the difference of the axes to the major axis, is about $\frac{1}{300}$. Now the axis of revolution is inclined at an angle of about 23° 28′ to the axis of the ecliptic: and, consequently, there is a small ring of protuberant matter, or meniscus, above the sphere whose diameter is the minor axis, inclined at the above angle to the ecliptic, and divided into two equal parts by it. Imagine this ring of matter to revolve independently of the earth, and in the same time, and to fix the ideas, imagine the state of things at the summer or winter solstice, when the angular distance of the sun from the earth's equator is greatest. In this case, the effect of the sun's disturbing action (that is, the difference between the pulling force on the centre of the earth and on any particle of the meniscus) will be such as to bring it towards the ecliptic, and, if the motion of revolution were to cease, it would ultimately pull it into coincidence with that plane; but, on account of its revolution, the effect will be so far modified that the inclination in a whole revolution will remain unaltered, but the points N N′, where the plane E Q cuts the ecliptic, will regress. In fact, each point being pulled towards the ecliptic,

will reach it sooner than it would otherwise have done, or the motion of the line N N′ will be in the contrary direction to that of the motion of revolution. Now in the case of nature, this meniscus is solidly joined to the earth, and



therefore whatever motion is given to the ring must be communicated to the whole body; and thus, on account of the large mass of the earth which the ring has to drag with it, the motion will be exceedingly diminished. The effect, however, will be the same; that is, the points of intersection of the ecliptic and equator will upon the whole retrograde. If we consider what will be the state of things at the equinoxes, we shall readily see that the sun being then in the plane of the earth's equator, and consequently there being no force to pull the protuberant matter out of its natural position, the retrogradation will be nothing; and in the same manner, that for all times intermediate to the equinoxes and solstices, the effect will be also intermediate.*

111. Thus far we have considered only the effect of the disturbing action of the sun, which, on account of his immense distance, produces but a small part of the observed retrogradation. The moon, though her mass is not $\frac{1}{30,000,000}$ part of the sun, produces, by being so near to us, a much more serious effect. It is very evident that her action will be of the same character as that of the sun, though the changes will be very much greater, on account of her wider excursions above and below the equator, and also because the plane of her own orbit is not fixed upon the ecliptic, but itself retrogrades, so as to go round the whole circle in a backward direction in the space of about 18¾ years. Now, during half of this period, the position of the nodes of the lunar orbit is such that its plane is but little inclined to the earth's equator, and therefore the precessional effect will be small; but during the remaining half, the inclination is considerably greater, and the lunar precession becomes thus proportionally

* See *Airy's Ipswich Lectures*, page 158.

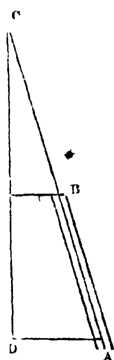
larger. These inequalities of precession, caused separately by the sun and moon, are called *solar* and *lunar* nutation, on account of the irregular motions which they produce in the poles of the heavens, and, consequently, in the polar distances of stars. The first evidently has for its period a solar year, or time of revolution of the earth round the sun; the second depends upon the place of the moon in her orbit, and also upon the time of revolution of her nodes, and therefore requires for its expression two terms, the one depending upon the moon's longitude, and the other upon the longitude of the node.

112. In all that has been said thus far, it has been assumed that the *plane* of the ecliptic is absolutely fixed in space, but even this is not the case, since the relative orbit of the sun and earth suffers also disturbances by the action of the larger planets, which require also to be taken into account. At present we are not in a condition to enter more largely upon this subject, and the general explanation of precession is not affected by it.

113. By application of corrections derived from the theory of precession and nutation, we are enabled now to represent the places of the stars for any epoch, as correctly as if the points of reference were absolutely fixed; yet we have still one correction to make, depending upon a phenomenon of a totally different character, we mean the phenomenon of the aberration of light. Roëmer discovered, by means of the eclipses of Jupiter's satellites, when observed at different distances of Jupiter from the earth, that light was propagated with an immense, though measurable velocity, so as to describe the radius of the earth's orbit in about 8½ minutes; but it was reserved for Bradley to show by observation, that this progressive motion of light, combined with the motion of the earth in her orbit, would produce an apparent displacement of every object in the heavens; throwing it apparently a little behind the place which it ought, according to theory, to occupy. The casual experiment by which Dr. Bradley was first led to the idea of the theory of aberration is instructive, not only in showing the process by which a great mind arrived at the explanation of this most important phenomenon, but as giving a really simple and intelligible elucidation of it. He was in a wherry on the Thames, which had a vane for observing the direction of the wind, at the top of the mast. The boat being stationary, he observed by this vane the direction of the wind, but it appeared to him, that when the

boat was again in motion the direction immediately changed. The boatmen were themselves familiar with the phenomenon, but were unable to explain it; and by reflection, Bradley was soon enabled to see that the change of direction must be not real, but apparent, and depending upon the unthought-of motion of the observer. Several other illustrations may be given, of which the most usual is that of the direction of rain-drops falling upon a person as he walks. If the drops fall vertically, and he be walking with any considerable velocity, they will strike him on the face, as if they came towards him in an oblique direction. An ingenious illustration is given by Mr. Airy, in the Ipswich Lectures. Imagine the side of a ship in motion to be pierced by a shot from a battery which she is passing, which goes out at the other side. It is evident that the point of the off-side of the ship at which the shot goes out will be farther a-stern than the hole made on entering the other side; and the sailors, if they did not attend to the motion of the ship, would imagine that it had passed through in an oblique direction. We will take one more illustration, which will enable us to see still more clearly the law according to which the effect of *aberration* must be calculated.

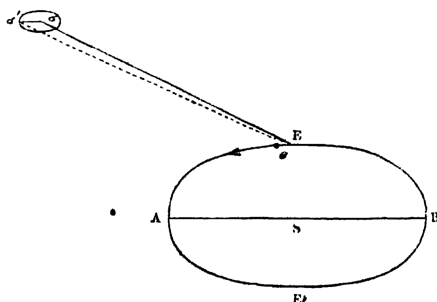
114. Let A B be a tube, making a fixed angle with the horizon, but carried forward horizontally in the direction A D, with a velocity represented by A D. Let now a body fall from C, with a velocity represented by C D, at the same time that the tube starts from A; then it is evident (by similar triangles) that when the top B of the tube has arrived at E, the body will also have fallen to E, or will enter at the top of it, and, the ratio of the velocities still remaining the same, will proceed within it, through its whole length, without striking the sides. If, also, an observer at A were looking up the tube, the body would appear to fall in the direction C B A, instead of the vertical direction. In this instance, then, the effect of *aberration* is the angle C, which may be calculated by the formula,



$$\text{Tan. C} = \frac{\text{D A}}{\text{C D}} = \frac{\text{velocity of observer.}}{\text{velocity of falling body.}}$$

Let us now apply this analogy to the case of an observer on

the earth's surface, looking at a star, the velocity of light being supposed known, and being in round numbers to that of the earth in its orbit nearly as 10,000 to 1, or as unity to the tangent of an arc of 20″·5. Let σ be the place of any



star, and E that of the earth at any time, describing its orbit round the sun, S, in the direction E A E′ B; E e, a small arc of the orbit described in the unit of time. Then, if a space be set off on E σ equal to that traversed by light in the same time, and the parallelogram be completed, of which these spaces are the sides, its diagonal will be in the direction E σ' of the apparent position of the star. The effect of this displacement will be to make each star in the heavens apparently describe a small ellipse round the true place unaffected by aberration, and the axes of this ellipse will depend upon the position of the star with regard to the ecliptic; that is, on its latitude. If, for example, the star were in the ecliptic, the whole displacement would be in this plane, the effect on the latitude of the star being nothing; but, if the star were situated at the pole of the ecliptic, the curve described round its mean place would be a circle with an angular radius equal to 20″·5; and generally, the axes of the ellipse will be in the proportion of unity to the sine of the latitude.

115. Without going into details, for which, in fact, we have not space, the intelligent reader will readily understand, from the principles already laid down, that the apparent displacement which every star experiences with respect to the ecliptic can be calculated, and the position of that plane with regard to the equator being known, the small corrections in right ascension and polar distance can also be computed. Conversely, by judicious observations, continued throughout every period of the year, of stars situated near the pole of

the ecliptic, where the effect of aberration is the greatest possible, it is plain that the axes of the *aberration ellipse* will be found, and the constant quantity (or numerical multiplier necessary for the computation of its effect upon every star) of its expression will be found. It was by observations of the star γ Draconis, which for Greenwich passes the meridian very near the zenith, that Bradley detected this important phenomenon. The right ascension of this star is nearly 18^h, and its north polar distance nearly equal to the co-latitude of Greenwich. It lies, therefore, very nearly in the plane of the great circle joining the poles of the ecliptic and equator (called, technically, the solstitial colure), and about 15° from the pole of the ecliptic. If, then, the reader has carefully attended to the preceding explanation, he will see that when the earth is in the autumnal equinox, and therefore moving parallel to the colure, the effect of aberration will be apparently to raise the star above the ecliptic, and therefore to bring it nearer to the north pole of the equator by the quantity 20·5 × sin. 75° (= 20″·5 × sin. lat.); but that when the earth is at the vernal equinox, the effect will be to increase the distance from the pole by about that quantity. If, then, the observations of polar distance made at these periods be corrected for refraction, precession, and nutation, their difference will amount to about 41″ × sin. 75°, or to nearly 40″.

116. M. Struve, the Russian astronomer, by observations recently made, has definitely fixed the constant of aberration at 20″·445, which, he believes, does not differ 0″·011 from its true value; and hence the velocity of light is known with very great accuracy, and the time taken to traverse the mean radius of the earth's orbit will be $8^m\ 17^s·78$ of mean time.

117. The effect of aberration on the places of the planets differs from that on the places of the stars on account of their motion in their own orbits. On this account it happens that they are seen by rays coming from a point different from that which they occupy when the observation is made. It is evident, in such cases, that the effect of aberration referred to any planes whatever will depend on the *relative* motion of the earth and the planet, that is, on the geocentric motion of the planet, and on its distance from the earth. Let, for instance, D be the distance of the planet from the earth; then the time in which light traverses this space is $8^m\ 13^s \times \frac{D}{R}$ where R is the radius of

the earth's orbit, supposed circular. This time is technically called the *aberration-time*, and the geocentric motion of the planet being known, the space through which it will move in the above time, whether in right ascension or declination, is the correction for aberration. The most convenient way of computing the *apparent* places of the planets, that is, their places affected by aberration, is to subtract the *aberration-time* from the time for which their places are required, and to consider the remainder as the time for which the computations are to be made.

118. There is still one effect of aberration to be considered, viz. that arising from the earth's diurnal rotation on its axis from west to east. This effect is so small as not to be appreciable by observation, but it produces an error in the places of all objects observed with the transit-instrument (throwing them too far east, or causing them to come to the meridian too late) of the same kind as the error of collimation, which is necessary to be taken into account. Its amount is easily calculated.

119 Let Greenwich be the station of observation, in north latitude 51° 29′; then the space through which the earth's equator is carried in the time t^s will be

$$\frac{t \times 2\,r \times 3{\cdot}14159}{24 \times 60 \times 60} = r\,t \times \frac{2}{27500} \text{ nearly.}$$

where r = the earth's equatorial radius; and the space through which the parallel at Greenwich is carried in t^s is

$$2\,r\,t \times \frac{\cos. 51^\circ\, 29'}{27500}.$$

but the space through which light moves in t^s

$$= \frac{t}{493} \times \text{R (where R = radius of earth's orbit).}$$

Hence,

$$\frac{\text{velocity of rotation at Greenwich}}{\text{velocity of light}}$$

$$= \frac{2\,r}{\text{R}} \times \frac{493}{27500} \times \cos. 51^\circ\, 29'$$

$$= \frac{8000}{95000000} \times \frac{493}{27500} \times \cos. 51^\circ\, 29' \text{ nearly.}$$

$$= \frac{1}{1063600}, \text{ nearly.}$$

120. This represents the tangent of the angle of aberration in this instance, and the corresponding arc is 0″·194. Every star, therefore, comes to the meridian too late by the time of its motion over this arc, and requires a correction in time $= - \frac{0 \cdot 194}{15 \sin. \text{N.P.D.}}$ The error itself is, however, in practice incorporated with the error of collimation.

121. We are now enabled by the application of the corrections due to refraction, parallax, precession, nutation, and aberration, to represent the places of the stars as they would be observed by a spectator placed at the earth's centre, supposed fixed in space, with no atmosphere causing displacement, and referred to fixed points and a fixed plane of reference. It is evident, however, that the corrections required for precession, nutation, and aberration will be computed from formulæ necessarily of some complexity. We are indebted to the illustrious Bessel, in the first place, for reducing the formulæ to a shape admitting of easy computation, in every case, by separating the terms of their expressions into double sets of factors, the one set containing only such quantities as depend on the time, such as the date of the observation, the position of the node of the lunar orbit, and the places of the sun and moon, on which the nutation and aberration depend; the other set involving only such quantities as depend on, or are functions of, the star's right ascension and declination.

122. The logarithms of the first set of quantities are given in the Nautical Almanac for every day of the year, under the designations A, B, C, D, and the large catalogue of stars recently published under the authority of the British Association gives for every star included in it the values of the logarithms of the second set of quantities for both R. A. and N. P. D., under the designations a, b, c, d and a', b', c', d'. The whole correction to be applied to the mean place of a star to obtain its apparent place for any day required is—

For Right Ascension . $\text{A}\,a + \text{B}\,b + \text{C}\,c + \text{D}\,d$
For N. P. D. . . . . $\text{A}\,a' + \text{B}\,b' + \text{C}\,c' + \text{D}\,d'$

The Astronomer Royal, Mr. Airy, has recently put these formulæ in a shape still better adapted for the use of ordinary computers, by altering them so that every term shall be positive, or so that the computer shall have no

regard to the signs of the expression until he arrives at the end of the process, where the correct sign of the correction results from the introduction of a constant which is subtracted. The general formula equivalent to those given above is, then, $E\,e + F\,f + G\,g + H\,h + L\,l - 300$.*

CHAPTER IV.

OF THE MOTION OF THE SUN IN THE ECLIPTIC, ETC.

123. In the preceding chapter it has been our object to bring together, so as to exhibit in one point of view, all the corrections which are necessary to be made before the true place of a celestial object can be found from observations made at the surface of the earth. In doing this, however, we have been obliged to anticipate in a slight degree, or to assume the reader's acquaintance with, a few of the leading facts of the solar motion, as well as some familiarity with the plane in which that motion is apparently performed. We have in the same manner assumed, in some degree, his familiarity with the motion of the moon round the earth. But, in fact, all we have required to be admitted is, that the sun *apparently* moves round the earth, or the earth *really* moves round the sun, in a nearly circular orbit, described in a plane, sensibly fixed, called the ecliptic. We have also assumed in like manner for the moon, that she revolves round the earth in a nearly circular orbit; but that the plane of her orbit, which has a small inclination of about 5° to the ecliptic, has a retrograde motion on the ecliptic by which it is carried completely round the circle in the space of nearly nineteen years.

124. It will be our object in this chapter and the following to treat of the motions of these luminaries a little more

* We would recommend the more advanced student not to rest satisfied till he has made himself thoroughly familiar with the admirable elucidation of these formulæ, given by Mr. Baily, in the Introduction to the British Association Catalogue, as also with the account of Mr. Airy's transformation of the formulæ in the Notices of the Royal Astronomical Society.

specifically, and at the same time to introduce such remarks on the theory of gravitation as will enable the reader to obtain an idea, however vague and imperfect, of the laws according to which the orbits are described, and of the perturbations to which they are subjected. In the present chapter we confine ourselves to the investigation of the solar orbit; to the consideration of the various measures of time used by astronomers; and, finally, to an account of the knowledge which has been acquired concerning the physical constitution of the sun.

125. The portion of the heavens in which the sun's *apparent* orbit is performed was called by the ancients the *zodiac;* and the great circle formed by the intersection of the plane of his orbit with the sphere of the heavens was divided by them into twelve equal portions or signs, to which were given, from the constellations through which the sun passes, the following names:—

Aries	Leo	Sagittarius
Taurus	Virgo	Capricornus
Gemini	Libra	Aquarius
Cancer	Scorpio	Pisces.

126. About 4000 years ago, the position of the vernal equinox, that is, of one of the points of intersection of the ecliptic or sun's path with the equator, coincided with the constellation Aries; but on account of the precession of the equinoxes, this point is now in the constellation Pisces. It is, however, customary with astronomers still to call the point of the spring equinox the *first point of Aries.* No confusion arises from the term, but great inconvenience is frequently experienced by an attempted change of nomenclature.

127. We have already, without alluding to any theory of the solar motion, shown how the mean distance of the sun, or the solar parallax, has been found by observations of the last transit of Venus across the disc, but we have not made any reference to the variations of his distance from the earth in the annual circuit. Now, if our readers would take the trouble to consult the Greenwich observations for any one year, and look at all the measures of the sun's diameter which have been made with the mural circle, they will not fail to perceive that in the course of the year the variations of the measures are far greater than can be attributable to errors of the observations, though,

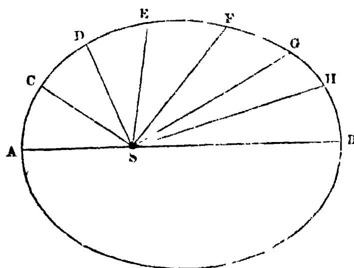
from the fluttering, badly-defined nature of the borders of the disc, these frequently amount to several seconds. If, however, any one would give himself the trouble to take the mean or average of the measures of the diameters on several neighbouring days, and thus divide the whole series into groups, he would observe a very conspicuous law in their variation. For example, he would find the measures smallest about Midsummer, and largest at the end of December, and between these times he would find a steady, uniform increase; while from January to the end of June, the decrease would be equally uniform. Now the object observed being the same, this proves that the distance of the sun is least in winter, and greatest in summer, and that the variations of increase and decrease are tolerably uniform.

128. A tolerable notion would thus be formed of the general shape of the orbit. It would be found to be not quite circular, but the extreme variations of distance would show that it does not differ much from a circle. In the year 1851, for example, the largest diameter given in the "Nautical Almanac" is that for the end of December, viz. 32′ 34″·6, and the smallest is that for the end of June, viz. 31′ 30″·0. The difference of these compared with the mean diameter is about $\frac{1}{31}$ part of the whole; and this, therefore, is the proportion in which the greatest distance of the sun from the earth exceeds the least. We might thus suppose, without further reasoning, that the earth moves round the sun, or the sun apparently round the earth, in an orbit slightly oval or elliptical; and a trial of this theory with observations of the diameters would show whether an exact ellipse is described.

129. It was, in fact, proved by Kepler, about three centuries ago, that not only the earth, but all the large planets describe ellipses about the sun, that body being in one of the foci of each of the ellipses. This law of the planetary motions is known by the name of *Kepler's first law.* The excentricities of these ellipses are different, and appear to be totally independent of each other; the planes of their orbits are also different, but are all included within a small angular distance from the ecliptic (about 10°), called the zodiacal limits; and, finally, the directions of their major axes are different. Still, all the orbits are ellipses described in the same direction in planes round the sun as a common centre.

130. Again, if we were to proceed really to construct the

curve of the solar orbit by assuming, as in the figure, the point S as the focus round which the sun appears to move, and, after calculating the sun's longitudes, from the observed right ascensions and declinations at times corresponding to those at which the diameters or proportional distances were observed, laying off angles C S D, D S E, E S F,



&c., equal to these differences, and lines C S, D S, E S, &c., inversely proportional to the observed diameters, we should not only find that the curve was sensibly elliptical, but that another law of the solar motion would become evident. We should find, in fact, that the velocity of the sun when near his shortest distance (*perigee*) S A is considerably greater than when near the greatest distance (*apogee*) S B, and that, being greatest at A, the velocity continually decreases till he arrives at B, when it is least; it then increases again during the remainder of the year till it is greatest at A. By comparing, for instance, the sun's increase of longitude in equal small times when near A and when near B (which gives the measure of angular velocity), we should find that the velocity at A would exceed that at B by about $\frac{1}{15}$ part, while the distance S B exceeds S A by about $\frac{1}{30}$ part. For instance, taking numbers from the "Nautical Almanac for 1851," we find that the daily increase of the sun's longitude at the end of December is about 1° 1′ 9″, but that in June it is about 57′ 13″. The difference of these is 3′ 56″, which, compared with the mean daily increase 59′ 11″, give the fraction $\frac{236}{3551}$, or $\frac{1}{15}$, corresponding to the ratio which has been previously assumed. The same is true in every other part of the orbit, that is, the increase of velocity is twice as great as the decrease of distance. Now, we might readily prove from this that the sun's angular velocity must be generally inversely proportionate to the square of the distance. For, let r be any distance from the earth, and v the corresponding angular velocity, then if v varies $\frac{1}{r^2}$, $r^2 \times v$ must be a constant quantity. Let now $r + x$ and $v + y$ be the radius vector and velocity at another time of the year. Then

the value of $(r + x)^2 \times (v + y)$ must be equal to $r^2 v$, because this quantity does not vary by the corresponding variations of r and v.

Hence, $$r^2 v + 2r v x + r^2 y = r^2 v$$

neglecting the small quantities $x^2 y$, $2 r x y$, $x^2 v$.

Hence, $$2 v x = - r y\text{; or, } \frac{y}{v} = - \frac{2x}{r}$$

That is, the increase of v compared with v is twice as great as the decrease of r compared with r. Conversely we may assume, that if the latter proposition be true, the quantity $r^2 v$ must be constant; or the velocity varies in the inverse proportion to the square of the distance or radius vector. Now this law, which belongs to all the planetary motions as well as to the sun, was also discovered by Kepler, and under the name of the *equable description of areas* is known as Kepler's second law. It may be distinctly announced as follows:—The areas swept out by the radii vectores of the planets (that is, by the lines drawn from them to the sun) are proportional to the times of their description; or, equal areas are described in equal times.

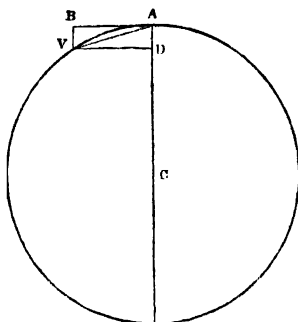
131. It will be well, in this place, to enunciate Kepler's third law of the planetary motions, though we are rather anticipating. He found, by a most laborious comparison of the planets' distances from the sun with their times of revolution. that a very simple and general law of connexion subsisted between them. The law simply enunciated is this, "that the squares of their periodic times of revolution are to each other as the cubes of their mean distances from the sun or the semi-axes major of their orbits." This is the law which we have before adverted to in the discussion of the solar parallax. We see by it that if the distance from the sun of any one planet be known, the distances of all the others can be found by simple proportion. Suppose, for example, that the mean distance of Mars be a, and its observed time of revolution t. Let the time of revolution of the earth round the sun be T and the mean distance required x. Then, by Kepler's third law, $\frac{x^3}{a^3} = \frac{T^2}{t^2}$

Hence, $$x = \left(\frac{T}{t}\right)^{\frac{2}{3}} \times a.$$

And the same rule will apply to any of the other planets.

132. Now the whole of the above laws are simple, mechanical consequences from the universal law of gravitation, to which we will now devote a few words. The sun, considered as a spherical body, exerts precisely the same influence upon a planet, considered as a spherical body, as if the whole mass of the sun and of the planet was concentrated at the centre of each. If this be assumed (the proof is by no means elementary or easy, though it *has* been proved by mathematicians), the application of the theory is simple. A point or centre of force of a certain energy attracts towards it another point moving at a given time with a definite velocity, in a direction *not* exactly *towards* nor exactly *from* the centre, with a force inversely proportional to the square of the distance; what curve will the body describe in consequence? This is the problem to be resolved, and Newton first demonstrated, and other mathematicians have since proved the same in various ways, that in all cases whatever the attracted body will describe a conic section, that is, a curve lying in one plane which is either a circle, an ellipse, a parabola, or an hyperbola. The particular curve described by the planet will depend upon the circumstances of the motion, that is, upon the direction of its motion with regard to the line joining it and the sun, and upon its velocity, at any known part of its orbit. For instance, if when the earth is moving with its greatest velocity, that is, when it is at the least distance from the sun and its direction of motion is at right angles to the radius vector, the velocity should be suddenly lessened in a small degree, it might describe a circle instead of its present elliptical orbit. If, again, the velocity were to be increased to a certain amount, it might never return towards the sun, but describe a parabola; and, finally, if the velocity were still farther increased, it would describe an hyperbola. These results are arrived at by an extension of the same kind of reasoning as is used in determining the laws regulating the fall or the motion of bodies near the surface of the earth, for which we would refer to the "Rudimentary Treatise on Mechanics." A stone thrown in any direction but the vertical will return to the earth after describing a parabolic curve. Now in this case it is manifestly impossible to give to the stone a sufficient velocity to enable it to clear the earth's surface, and, even if it were practicable to give it a sufficient initial velocity, the resistance of the air (varying as the square of the velocity) would so instantaneously and

enormously diminish it, as to render it useless. Also, the whole range of the stone is so small, compared with the earth's surface, that throughout it gravity may be supposed to act in parallel lines and with a constant amount of force. Hence the problem becomes very much modified. But, if we imagine the stone to be thrown from a *very* great height above the earth's surface, in an oblique direction to the line joining it and the earth's centre, it is still the *same* problem which we have to solve, only the data have become more complicated. The force, in this instance, does not act in parallel lines but towards a centre, and it is not a constant but a varying force, depending upon the distance from the centre of the earth. But, to fix the reader's ideas, we will take the simplest case of possible planetary motion, viz. that wherein the body, by the influence of a central force, is made to describe a circle round the centre, and we will find what the law of the force is in this instance.

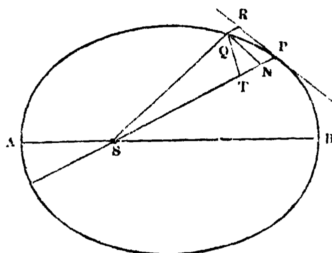


133. Suppose a body A to revolve in a circle round a centre of force C. At every point it is moving in the direction of the tangent at that point, and, if the force were suddenly to cease, it would go on, with a constant velocity, in that direction. Suppose, then, that when left to itself it would describe the space A B in a given small space of time; and suppose, also, that the force in the direction A C would draw it through the space A D in the same time. Then, if the parallelogram A B V D be completed, V will be the actual place of the body, which is, by hypothesis, in the circumference of the circle. Hence A D or B V $= \frac{AV^2}{2AC}$. If, now, A V and B V be indefinitely diminished, A V will become equal to the arc A V; and, if the given time be taken as the unit of time, will represent the velocity of the body in the curve. Also, B V, being the space through which the body is drawn in the unit of time, will represent the half of the force (F). Hence, if V be the velocity and R the radius of the circle, we have $F = \frac{V^2}{R}$, or, the velocities of bodies

describing circles, if the force be constant, will be as the square roots of the radii of the circles. For instance, if a stone attached to a string be whirled round so as *just* to be retained in a circle, as the string is lengthened the velocity in the curve will be increased in the duplicate proportion. But, if we wish to know how the angular velocities vary, putting a to represent it, we have $V = R \times a$; or $F = a^2 R$.

134. Hence, the tension of the string remaining the same, the radius varies inversely as the square of the angular velocity. But if the angular velocities be constant, or the circles be described in the same time, then the tension of the string will be increased in the simple proportion of the radius.

135. Similarly, if we wished to find according to what law of force a body might describe an ellipse round a centre of force situated in one of the foci, we should use similar reasoning.



136. Thus, let S be the centre of force; P the position of the body at any time, moving in the direction P R with a certain velocity denoted by P R; P N or R Q, the space through which the body would be drawn by the action of the force in the infinitesimally small time T; P R Q N being a parallelogram. Then, if F be the force which deflects the body through the space R Q, we have, by the second law of motion, $Q R = \frac{1}{2} F T^2$. But, if A be the constant area described in the unit of time, the area Q S P will be $A \times T$. This area is, however, ultimately equal to $\frac{1}{2}$ S P $\times$ Q T, Q T being perpendicular to S P. Hence $T = \frac{QT}{2A} \times SP$.

Hence F, which is ultimately equal to the limiting value of $\frac{2QR}{T^2}$

$$= 2QR \times \frac{4A^2}{QT^2 \times SP^2}$$

$$= \frac{8A^2}{SP^2} \times \text{limiting value of } \frac{QR}{QT^2}$$

137. To understand what is meant by the limiting value of $\frac{QR}{QT^2}$, imagine the point Q to approach towards P, and at length to reach it; then, though each of the quantities QR and QT^2, taken separately, ultimately vanishes, yet their ratio continually approaches nearer and nearer to some definite value, which can be found for any particular curve under consideration. For the ellipse this ratio is a constant quantity, that is, it is the same for every point of the curve, and hence the force varies as $\frac{1}{SP^2}$.

138. The foregoing considerations will show the nature of the investigations by means of which Newton traced his way to the true theory of the planetary motions. Assuming that the planets were kept in their orbits or made to describe certain curves in one plane by a force directed towards the sun, his object was first to find what must be the law of force by which this could be effected. Kepler had proved that the orbits were ellipses having the sun in one of the foci, and Newton easily proved, in the way we have explained, that the force must be inversely proportional to the square of the distance. He discussed likewise the laws of force for other curves, and other centres. For circles his discussion is nearly as we have given it; and, if an ellipse were described round the centre, he finds that the force must vary directly as the distance. It is evident, therefore, that, as far as the planetary motions are concerned, the force must be that of the inverse square of the distance, for no other law of force can be reconciled with Kepler's first law derived from observation.

139. Again, he found that for any law of central forces whatever, the areas described must be proportional to the times; and this latter fact being previously known as the result of observation, in the description of the orbits of the planets round the sun, it is plain that the force by which the planets are kept in their orbits is directed to the sun. This proposition was therefore to be proved before the laws of force round a fixed centre could be considered, and thus, in Newton's "Principia" it forms the first proposition, though we have taken it according to the order of Kepler's laws.

140. Lastly, Newton found that the time of the description of any elliptic orbit round a centre of force varying inversely to the square of the distance would vary in the sesquiplicate

ratio of the semi-axis major directly, and of the square root of the mass of the attracting body inversely. Thus, if T be the time of revolution, a the semi-axis major, and μ the mass, $T = \frac{2\pi a^{\frac{3}{2}}}{\sqrt{\mu}}$ where $\pi = 3{\cdot}14159$. This we can prove easily enough in the case of the circular orbit before discussed. Taking the equation $F = a^2 R$, let T be the periodic time, then $T \times a = 2\pi$, and $F = \frac{\mu}{R^2}$ (by hypothesis of the force varying according to the inverse square of the distance.)

Hence,
$$\frac{\mu}{R^2} = \frac{4\pi^2}{T^2} \times R.$$

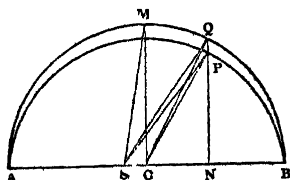
$$\text{or, } T^2 = \frac{4\pi^2}{\mu} \times R^3$$

$$T = \frac{2\pi R^{\frac{3}{2}}}{\sqrt{\mu}}$$

and this expression is equally true in the case of an elliptic orbit, and is, in fact, the symbolical enunciation of Kepler's third law.

141. Having thus discussed the simplest of the features of the theory of gravitation, and shown how it can be proved that Kepler's observed laws are some of the simplest consequences of it, we will proceed to show in what manner the motion of the sun in longitude can be calculated, and to exhibit the principles of the construction of the Solar Tables.

142. Let A P B be the half of the elliptic orbit of the earth described round the sun at S, a focus of the ellipse. Upon A B describe the semicircle A Q B. Let P be a position of the earth at any time, and draw the line Q P N perpendicular to A B. Join S Q, S P, C Q, C P. Imagine now a body to describe the circular orbit B Q A in the same periodic time that the earth describes its elliptic orbit, and let both start together from A, the *perihelion;* also when this body is at M, let the earth be at P; the latter having got beyond the



former on account of its greater angular motion near the perihelion. The angle A C M is called the *mean anomaly* and A S E the *true anomaly*. If, then, the relation between those angles can be expressed, that is, if the *true* anomaly can be expressed in terms of the mean anomaly, the earth's real motion or the sun's apparent motion in longitude is known. For, suppose the time taken by the earth in moving from A till it returns to it again be called P, and the time of describing the angle A C M be called T; then, since the circle A M B is described uniformly, we have (if angle A C M = M),

$$M = \frac{T}{P} \times 360^\circ$$

and, therefore, the true longitude, if it can be expressed in terms of M, can be calculated. Now, this can be done by means of an infinite series in terms of the excentricity of the orbit, but it is usual with astronomers to use an intermediate angle A C Q, called the *excentric anomaly*, to effect this development.

143. In fact, the mean anomaly can be expressed very simply in terms of the excentric anomaly, and the true anomaly can also be so expressed, and, for the benefit of such of our readers as are conversant with trigonometry, we will give the equations connecting these three quantities, as our reasoning will thus be rendered clearer.

If m, u, and v be respectively the *mean*, the *excentric*, and the *true* anomaly, and e the excentricity of the orbit, then

$$m = u - e \sin. u$$

$$\text{and } \tan. \tfrac{1}{2} v = \sqrt{\frac{1+e}{1-e}} \tan. \tfrac{1}{2} u.$$

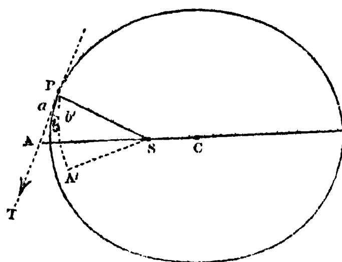
144. We should be going too far beyond the limits of an elementary treatise if we attempted to show how, by means of the above equations, v is expressed in terms of m. It is sufficient to say that it can be so expressed in a series whose first term is m, and that the other terms, involving powers of e and sines of multiples of m, are easily calculable, because e is for all the planetary orbits very small. We get then an equation of this shape—

$$v = m + E$$

and this latter term is calculated for each planet in extensive tables with the *mean anomaly* for the argument, and is

called the *equation of the centre*. This is the first of the *equations* or *inequalities* in the computation of the longitude of the sun or a planet, which is to be taken into account, and, if the orbits of the planets were strictly elliptical, it would be the only one. But, as we shall see presently, the action of the other large planets disturbs not only the plane of the orbit of any one of them, but also the position of the body in the orbit, and other corrections, called *perturbations*, must be applied to its longitude, computed from the elliptic theory, before the *true* longitude can be obtained by addition of the longitude of the perihelion or aphelion to the true anomaly.

145. There is one disturbance of the solar orbit which we shall have occasion to mention before proceeding further, and that is the progressive motion of the *apogee* or *perigee*, or the points of greatest and least distance from the earth, and as this is common to the other planets, we will take this opportunity of saying something of its physical cause.



146. Let P be a position of the earth, or other planet near the perihelion A, and imagine that at P the force in the direction P S is suddenly increased by a small quantity, which soon afterwards ceases. Take P a in the direction of the motion at P to represent the velocity, and a b parallel to P S to represent the deflecting force at P in the undisturbed orbit. Take also a b' to represent the increased deflecting force. Then it is plain that the planet will be made to move in a curve such as P b' A$'$ lying within P b, and making at P a more acute angle than S P T with S P. With regard to its point of least distance then, or *apse* A, it will be as it were thrown back, since the angle S P T has been continually increasing in the progress of the planet from B to A; or, the place remaining the same, the *apse* is thrown forward as to A$'$, or, in other words, the *apse* progresses, or goes forward in the direction of the motion.

147. Again, soon after the planet has passed perihelion, if we suppose the deflecting force to be suddenly increased, the direction of its motion, which then makes an obtuse angle

with the radius vector, will be changed, so as to make an angle more nearly equal to a right angle; that is, the planet will be nearer to the apse of the disturbed orbit, and the apse still progresses. Hence, when the force drawing a planet towards the sun is increased near the perihelion position, the apse progresses; and, by similar reasoning, if the force were diminished, the apse would *regress*, or go backwards, contrary to the direction of the motion.

148. Again, if the planet be near to, and approaching the aphelion B, the tendency of an increase of force will be to make the direction of motion more nearly perpendicular to the radius vector, or to bring the apse of the orbit backwards, or make it regress; and similarly, after the aphelion has been passed, the direction of motion will, by the disturbing force, make a more acute angle with the radius vector, or the apse will still *regress*. Finally, near the aphelion, if the gravitating force be diminished, the apse will progress.*

149. Since, then, a disturbing force acting towards the sun produces opposite effects at the perihelion and aphelion, there must be some intermediate points of the orbit where the effect of a slight disturbance acting towards or from the sun, will be nothing; and other points where a nice determination is necessary to determine what will be the effect. At present we are only concerned with the general explanation of the phenomenon, and with the general effect on the relative orbit of the sun and earth. By the action of the planets the radial force tending to draw the earth towards the sun is sometimes increased and sometimes diminished, but the mean, or average effect, is to make the apse progress by a minute quantity amounting to about 12″ in a year.

150. This motion has been detected by comparison of the longitude of the aphelion calculated from observations of the sun at widely distant periods. The place of the perihelion or aphelion is, in fact, one of those elements of a planet's orbit which must be known before its place for an assigned epoch can be calculated. If the reader remembers the process which was sketched in (144) for finding the longitude of the sun, he will see that the elements really involved are—

1. The mean anomaly, which depends on the mean motion, or the semi-axis major of the orbit.
2. The excentricity of the elliptic orbit.
3. The longitude of the perihelion or aphelion.

* See Airy's Gravitation, page 41.

151. In the case of the sun, which moves in the ecliptic, these are sufficient to determine the true longitude ; but, in the case of other planets, two other elements are necessary, viz.—those which define the position of the planes of their orbits with regard to the ecliptic. These are, the inclination of the plane of the orbit to the ecliptic and longitude of the nodes—that is, of the points where the orbit cuts the ecliptic. The elements that are necessary, then, to determine the position of a body are, the mean distance, the excentricity, the longitude of aphelion or perihelion, the longitude of the node, and the inclination of the orbit to the ecliptic, and finally, the mean longitude at a given time.

152. To return, then, to the consideration of the motion of the perigee of the solar orbit. By certain assumed values of these elements, derived originally from observation, the longitudes of the sun can be calculated, and compared with longitudes derived from observed right ascensions and North-polar distances. Hence, the errors of the calculated longitudes can be found, and can be again expressed in terms of the errors of the elements (for it is evident that the whole error in longitude is made up of the errors produced by each of the erroneous elements taken separately) ; thence a series of equations can be formed, from which the corrections of the elements can be obtained, and a corrected value of the longitude of the perigee or apogee can be found. Imagine this to be done for two epochs as distant as possible, that is, as distant as will allow of two sets of trustworthy observations. Two distinct values of the longitude of the perigee or apogee will thus be obtained, and the difference of these divided by the number of years, separating the observations, will give the apparent annual motion with regard to the equinox, supposed fixed. Correcting this for the precession of the equinoxes (See Chapter III.), we could obtain the real *progression of the apse* of the solar orbit, which we have said amounts to about 12″ per year.

153. Having thus treated of the theory of the solar motion, we will proceed to apply it to the explanation of the various measures of time used in civil life and by astronomers. For the purposes of common life, since our daily business is regulated by the times of the rising, setting, and culmination of the sun; since also he regulates the return of the seasons in the same recurring order ; it is evident that our smaller as well as our larger measures of time, that is, our days and our years, must be regulated by this luminary. Astronomically

speaking, this method of measuring time by the sun has its defects, on account of the irregularity of his motion with regard to the equator, but still no other can be found which would so well answer all those common needs which render requisite an artificial measure; and hence astronomers have devised means of obviating the inconvenience of the irregular motion of the sun, and of still employing him for the purpose. It is not absolutely requisite that the clocks and the chronometers which are used to give ordinary indications of the lapse of time, and which *must* go on the whole regularly or equably, (at least it is manifestly impossible to make them imitate or represent any periodical inequalities of motion,) should denote exactly the position of the real sun with regard to the equator. If we could always find the position of an imaginary sun moving uniformly in the equator, and on the whole never deviating far on one side or the other from the real sun, it is plain that such an imaginary body would answer almost equally well. The mornings, that is, the interval from sunrise to noon, might sometimes be a few minutes longer or shorter than the evenings, or the intervals from noon to sunset; and so we might (as we actually do) have very short afternoons before Christmas, when we should be glad of a longer duration of daylight, but this is a very slight inconvenience contrasted with the advantages of an equable standard agreeing so closely with the sun.

154. The *mean* sun, then, describes the equator with the sun's mean motion or longitude, that is, it is supposed to describe on the equator daily an arc of 59′ 8″·33, with a uniform motion; while the *real* sun describes the ecliptic, with a motion sensibly irregular, on account of the elliptic orbit. Hence, his motion, measured along the equator, is irregular on two accounts,—first, because his place being referred to the equator by a perpendicular arc drawn to it, his distances from the equinox, measured along the ecliptic and equator, are not equal; and, secondly, on account of the unequal motion in the ecliptic. From both these causes conjoined the true sun sometimes passes the meridian before and sometimes behind the mean sun, that is, a dial which gives the true motion of the sun referred to the equator will sometimes point to 12 o'clock before and sometimes after mean noon. In fact, the real and the fictitious sun cross each other four times in the year. For example, in the year 1851, the mean noon coincided with the apparent noon, as shown

by the sun's transit on April 15, June 15, September 1, and December 25. The difference in time for any day between the transits of the real and fictitious sun, is called the *equation* of time, and is given in the first page of each month of the "Nautical Almanac."

155. Astronomers are, however, in the habit of using another measure of time, viz. *sidereal* time. A *sidereal* day is the interval between the departure and the return of a star to the same meridian, as the solar day is the interval between the departure and return of the mean or the real sun. A clock used in connexion with the transit instrument is set so that its index shall denote $0^h\ 0^m\ 0^s$ (or nearly so) when the first point of Aries is on the meridian, and the sidereal time commonly used by astronomers denotes the hour angle (west) of this point, and is therefore affected by the *equation of the equinoxes*. As the effect of nutation causes the motion of the equinoxes to be irregular, sidereal time is not, strictly speaking, a uniformly increasing quantity, and might be distinguished into *mean* and *apparent*, in the same manner as solar time. But the smallness of the fluctuations arising from this cause (being only $2^s{\cdot}3$ in a revolution of the moon's nodes, or 19 years), makes this refinement unnecessary, and therefore, in practice, *sidereal* noon is the instant when the true vernal equinox is on the meridian, and a sidereal day is, in practice, the interval between two successive returns of the equinox to the meridian.

156. Since the mean sun moves in the equator from west to east, contrary to the diurnal motion, with a daily motion of $59'\ 8''{\cdot}33$, or, in time, of $3^m\ 56^s{\cdot}555$, the mean solar day will be longer than the sidereal day by this quantity; and, therefore, 24^h of mean solar time are equivalent to $24^h\ 3^m\ 56^s{\cdot}555$ of sidereal time; or, the ratio of a sidereal day to a mean solar day is as

$$1 : 1 + \frac{3^m\ 56^s{\cdot}555}{24^h}, \text{ or as } 1 : 1{\cdot}002738.$$

157. We will now proceed to the explanation of the larger divisions of time. In very early ages some nations reckoned their years by synodical periods of the moon. A *lunar* year with them was the interval between successive conjunctions of the moon, the time of new moon admitting of tolerably accurate observation. But the greater number of civilised nations have taken a revolution of the sun as the unit for

their year, though it required some progress in astronomical knowledge before this could be rendered an accurate standard. It is evident that the interval of time that was wanted was that in which the sun after leaving the equinox returns to it again, for on this the return of the seasons depends. This measure of time is aptly called the *tropical year*. The *sidereal year* is in like manner the time between the sun's departure from and return to a fixed star or other fixed point in the heavens. Finally, the *anomalistic year* is the interval between the departure of the sun from the perigee or apogee of his orbit till his return to it again.

158. Now, as the equinox goes backward annually to meet the sun by a quantity amounting to $50''\cdot224$, the sidereal year is longer than the tropical year in the proportion of

$$360^\circ : 360^\circ - 50''\cdot224, \text{ or as } 1 : 1 - \frac{50\cdot224}{1296000}; \text{ or, nearly as}$$

$$1 + \frac{50\cdot224}{1296000} : 1; \text{ or as, } 1\cdot00003875 : 1$$

But, according to Bessel, the tropical year (which is subject to a very slow variation) was, at the beginning of this century, equal to $365^d\ 5^h\ 48^m\ 47^s\cdot819$, the days being of course mean solar days. Hence a sidereal year is longer than this by $20^m\ 22^s\cdot9304$, or is equal to $365^d\ 6^h\ 9^m\ 10^s\cdot749$. This result the reader may verify for himself.

159. Again, the sidereal year is shorter than the anomalistic year in the proportion of $1 : 1 + \frac{12}{1296000}$; or of $1 : 1\cdot00000926$, and the reader may readily calculate its length.

160. We have not much space to devote to the consideration of the formation of the *calendar*, or the means taken to prevent the accumulation of error in the course of years on account of the hours, minutes, and seconds, which, over and above the 365 days, make up the tropical year; but we can give a few words of explanation. If we were to assume the year to consist of 365 days instead of its exact amount, an error amounting to very nearly a day would be entailed every four years, and this in a century would throw the minor divisions very nearly a month out with regard to the seasons. It was, in fact, this accumulated error, amounting at the time to ninety days, which led to the Julian correction, so called because of its celebrated

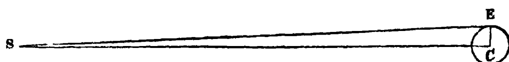
author, Julius Cæsar, who carried it into effect about forty-five years before the Christian era, with the assistance of the Egyptian astronomer Sosygenes. This correction consists in intercalating, or adding, a day in February every fourth year; that is, making every fourth year to consist of 366 days instead of 365 days, and the corresponding February to consist of twenty-nine days instead of twenty-eight days. The year thus augmented was called by the Romans *bissextile*, and is now here familiarly called leap-year. But it is evident that even now the correction is imperfect. It is based on the supposition that the average year consists of 365 days 6 hours, which exceeds its true amount by 11 minutes 12 seconds. The correction is therefore too great by nearly $\frac{11 \cdot 2}{360}$ or by $\frac{1}{32}$ part nearly. Hence arose the Gregorian correction devised by Pope Gregory XIII., in the year 1582, which was adopted in Catholic countries in the sixteenth century, but not in England till the year 1752, when the calendar which was wrong by 11 days was corrected, and the New Style commenced.

161. The Gregorian correction consists in omitting three leap-years out of one hundred; that is, in omitting one leap-year at the beginning of three successive centuries, and retaining it at the commencement of the fourth, and is equivalent to subtracting $\frac{3}{100}$ part from the Julian correction to the year. Now we saw that the quantity really wanted is $\frac{1}{32}$ part, and the difference is so small that it will not cause an error of a day in four thousand years. The leap-years are those which can be divided by four without remainder (thus 1852 is a leap-year), and the remainder after dividing any other year by four will indicate the number of years since the last leap-year. The Gregorian correction omits the intercalation of the day in the secular years 1700, 1800, 1900, but retains it for the next secular year, 2000, and so on, repeating the omission of the intercalation every fourth century: and the rule for determining which of the years commencing centuries are bissextile and which are not, is analogous to the former; omitting the last two cyphers, if the remaining number is divisible by four the year is bissextile, but if it be not so divisible the year *is not* bissextile.*

* For a very valuable article on the Calendar and the Theory of Chronological Epochs, see Sir J. Herschel's Outlines of Astronomy. Part iv. chap. xviii.

162. We have now concluded our remarks on the theory of the apparent motion of the sun, and its application to the measure of time, and it remains that we give a short account of what is known of the physical constitution of this wonderful body, and its relation to us with regard to our comfort and convenience.

163. Our attention is first drawn to its size and mass; and the principles already explained will enable the reader to see that we can directly measure these elements. The solar parallax has been accurately measured by means of the transits of Venus (See 90.), and amounts to 8″· 5776. This, the reader will remember, represents the angle which the semi-diameter of the earth would subtend at the centre of the sun, and from this the distance is easily deduced.



Thus, in the annexed figure, if S be the centre of the sun, and C that of the earth, C E the earth's equatorial semi-diameter,

Then, $SC = EC \times \text{cotan. } ESC.$
$= EC \times \text{cotan. } 8''\cdot5776.$

By calculation, the distance S C will be found to be 23,984 times the length of the earth's radius, or, in round numbers, to above 95,000,000 miles. Now the mean value of the sun's angular diameter as seen from the earth, is according to Bessel, 32′ 1″·8. Hence it is evident that the diameter, expressed in miles, will be 190,000,000 × tan, 16′ 0″·9, that is, to twice the distance multiplied by the angular semi-diameter. This gives for the actual diameter of this stupendous globe, expressed in English miles, the almost incredible value of 885,000 miles. It will perhaps assist the reader in forming an idea of the real size of the diameter of the sun, thus set down in figures, if we compare it with the dimensions of the lunar orbit. The mean distance of the moon, is about 240,000 miles. If then, we imagine the centre of the earth to coincide with that of the sun, the surface of the latter body would not only include the lunar orbit, but would extend almost as far beyond. The mass of this body is no less wonderful (our readers who have read the Rudimentary Treatise on Mechanics will be familiar with the astronomical meaning of the term *mass*). Since all

bodies in nature gravitate towards each other, the energy or force with which one body of the solar system attracts or draws any other is represented by this term, and if the matter of which the attracting body is composed be homogeneous, the mass would be proportioned to the quantity of matter, though this latter idea belongs more to bodies of small size, which we have occasion to compare on the surface of our own planet. However, in astronomy, the attracting force only of a planet on another is meant by the term, and this is measured, as we have had occasion to explain (133) by the space through which the attracted body is deflected in a given time, or by the curvature of its orbit. On this depends the time of revolution of the planet, and conversely the known times of revolution of planets which have satellites compared with the times of revolution of the satellites round their primaries, are used for determining the mass of the sun. Thus if M, m, and m' be the masses of the sun, the earth, and the moon, and A, and a, the semi-axes of the solar and lunar orbits, then if T and t represent the times of revolution of the earth round the sun and of the moon round the earth, that is a sidereal period of each of these luminaries ; we shall have

$$T^2 = \frac{4\pi^2 A^3}{M + m}, \text{ and } t^2 = \frac{4\pi^2 a^3}{m + m'}$$

Hence, $$\frac{M + m}{m + m'} = \frac{A^3}{a^3} \cdot \frac{t^2}{T^2}$$

Now, the radii of the lunar and solar orbits are in the proportion of 240,000 : 95,000,000, or as 1 : 400 nearly; and the revolution of the moon round the earth is to that of the earth round the sun in the proportion of 1 : 13·4 nearly.

Hence, $$\frac{M + m}{m + m'} = \frac{(400)^3}{(13{\cdot}4)^2} = 356{,}430 \text{ nearly.}$$

And as m is so small compared with M, we may safely neglect it in making this comparison.

Therefore, $$M = 356{,}430 \times (m + m')$$

that is, the mass of the sun is approximately three hundred and fifty-six thousand times the sum of the masses of the earth and moon.

164. To form some idea of the effect of this enormous dis-

proportion of masses, let us assume that $M = 356,430 \times m$, the mass of the moon being small, and try what will be the proportion of the pressure exerted on two individuals, one on the surface of the sun, and the other on that of the earth. The effects of the masses will be the same as if the same energies were exerted at the centres of the sun and earth, and will be inversely proportioned to the squares of the distances from the centres to the surfaces, that is to the squares of the radii. Let R and r be the radii of the sun and earth, P and p the pressures exerted on the same particle of matter at the surfaces,

Then, $$P = \frac{M}{R^2}, \text{ and } p = \frac{m}{r^2}$$

Hence, $$\frac{P}{p} = \frac{M}{m} \times \frac{r^2}{R^2}$$

$$= 356,430 \times \frac{(4000)^2}{(442,000)^2}$$

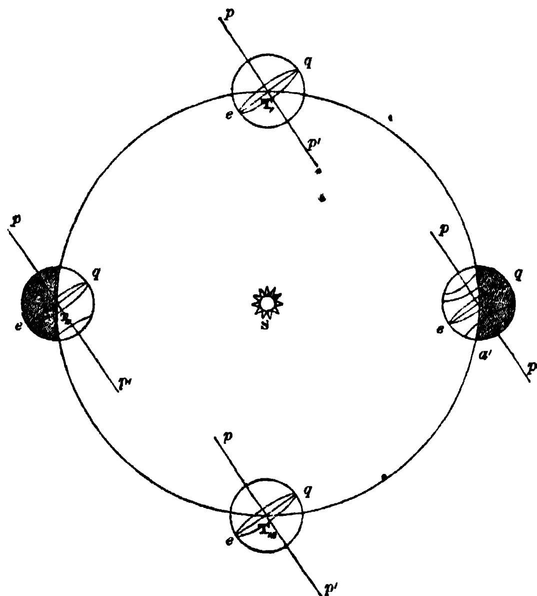
$$= \frac{356,430}{(110)^2} \text{ nearly}$$

$$= 29 \text{ nearly.}$$

Hence a man weighing 150 lb. at the earth's surface, would be, at the surface of the sun, pressed towards the centre with a force equivalent to 29 × 150 lb., or to about two tons. In fact he would be completely crushed by his own weight or gravitating tendency. The same reasoning will apply to some of the planets which have satellites, and the result is that there is no known planet but the earth, which is adapted for the muscular energy and temperament of man and the other animals, and hence we have a beautiful specimen of design and of the adaptation of the globe which we inhabit to the nature and constitution of its inhabitants.

165. The vicissitudes of the seasons are regulated by the elevation or depression of the sun in his annual circuit above or below the equator. Thus at midsummer his elevation above the equator is about 23°½, being equal to the obliquity, or inclination of the ecliptic to the equator. At midwinter he is depressed below the equator by the same quantity. Finally, at the vernal and autumnal equinoxes he is situated in the equator, being then at the points of intersection of the equator and ecliptic.

166. We will show by a diagram the effects produced on the length of the day and night by these circumstances. Let T, $T_{,}$, $T_{,,}$, $T_{,,,}$, represent four positions of the earth, corresponding to the summer solstice, the autumnal equinox, the winter solstice, and the vernal equinox, in its annual orbit round the sun, S; let *p p'* be the axis of rotation continuing always parallel to itself and let *eq* be the equator.



Then at T, the sun is above the equator by the angle S T *e*, or by the obliquity (23°½), and if we draw *a a'* perpendicular to S T, the shaded portion of the globe represents the part on which the sun's light cannot fall, or where night exists, and the unshaded portion represents the illuminated hemisphere, or the portion where day prevails. Hence by drawing small circles parallel to the equator, we observe that for all parts above *a*, that is in the northern arctic regions, there is no night, and that for all parts below *a'*, that is for the southern arctic regions, there is no day;—for any point above the equator, the days are longer than the nights by the greatest possible quantity;—and for any place in the

southern hemisphere the nights are longer than the days by the greatest possible quantity. All these phenomena will evidently be reversed at $T_{\prime\prime}$, or the winter solstice, where the north pole is turned away from the sun, and the south pole turned towards him. Finally, at $T_{\prime}$ and $T_{\prime\prime\prime}$, or at the equinoxes, the days and nights are equal throughout the globe, since the half of any circle of latitude is half in light and half in darkness.

167. With regard to the physical peculiarities of the surface of the sun, our knowledge is derived from telescopic observations. If the image of the disk be projected on a screen in a darkened chamber, or if it be viewed directly through a telescope of which the eye-piece is provided with coloured glasses for the protection of the eye, it will be generally found that the disk is not uniformly bright, and that in some places spots (*maculæ*) of absolute darkness occur upon it. These spots vary exceedingly at different times. At some periods large groups of these spots suddenly break out and continue for a considerable time, while at other times the disk is comparatively clear for a long period. They are exceedingly irregular in their form, but they all, when of any magnitude, agree in having the middle portion of intense blackness, while the margin is surrounded by a penumbra only partially shaded. If the attention be directed from day to day to any remarkable spot which has appeared near the sun's eastern borders, and its position be mapped down on a circle drawn on paper, it will be found to have a motion in an elliptic curve from east to west, increasing in rapidity as it approaches towards the centre, and becoming very slow near the western border. From such observations it becomes evident that the spots are not bodies revolving round the sun, but that they exist upon his surface and revolve with him. They can frequently be observed during at least one or two revolutions, and in rare instances they have been watched during several; but small ones frequently disappear in the course of a few days. Advantage has been taken of the durations of these revolutions for the determination of the time of rotation of the sun. The problem is difficult and not adapted for a popular treatise; and the reader may be referred to Sir J. Herschel's Outlines of Astronomy, for an excellent sketch of the method by which the time of rotation is deduced. We will content ourselves with simply recording the results. According to the best recorded determinations, the inclination of the sun's equator

to the ecliptic is about 7° 20′, the longitude of the ascending node being 80° 21′, and the period of rotation is 25^{d} 7^{h} 48^{m}.

The spots are generally confined to the neighbourhood of the sun's equator, and are never found in the polar regions; they are also frequently found arranged in the manner of belts parallel to the equator. These circumstances seem to indicate that they owe their origin to the rotation, and are produced by disturbances in the solar atmosphere occurring much in the same way as storms above the surface of our own planet. Sir William Herschel's hypothesis concerning their formation, which is based on these circumstances, is the only one which will bear the test of much examination. He supposes "luminous strata of the atmosphere to be sustained far above the level of the solid body by a transparent elastic medium, carrying on its upper surface (or rather at some considerably lower level within its depth) a cloudy stratum, which, being strongly illuminated from above, reflects a considerable portion of the light to our eyes, and forms a penumbra, while the solid body shaded by clouds, reflects none. The temporary removal of both the strata, but more of the upper than the lower, he supposes effected by powerful upward currents of the atmosphere, perhaps from spiracles in the body, or from local agitations."*

168. In connexion with the *maculæ*, or spots, another singular phenomenon is witnessed on the surface of the sun, which consists in strongly-marked curved or branching streaks or lines, brighter than the surface in the neighbourhood, called faculæ. They are most commonly seen near the borders of the disk, and are either in the neighbourhood of large spots or are precursors of their formation. They bear every appearance of a violent agitation in the luminous atmosphere or envelope of the sun.

169. Some additional and most interesting knowledge of the constitution of the solar atmosphere has been gained by the observations of total eclipses of the sun, and especially by the well organised series of observations which has been made along the line of totality of the recent eclipse of 1851, July 28. This eclipse was total for several countries in the North of Europe, and especially for Sweden, Norway, and parts of Prussia and Poland. Parties of observers, well

* Sir J. Herschel's Outlines, page 229.

furnished with instruments and with elaborate instructions for the observations of every phenomenon which former experience had suggested, were despatched from England, France, and Germany, to the various towns and stations (previously selected) which could ensure the full observation of the phenomena in all possible phases. The circumstances were exceedingly favourable, the moon being near perigee, and the eclipse occurring early in the afternoon, when the sun was high.

The results fully answered the expectations which were formed concerning it. All the observers agree in the main features of this grand and terrible phenomenon, while the variations, arising from their differences of locality, give much important information concerning some circumstances which could not have been obtained at any one station. As soon as the disk of the sun was completely hidden by the moon, a bright corona of white light, similar to that of an aureola or glory, was seen round the border of the moon, now appearing like an intensely black patch in the sky. Round the black circle of the moon, at irregular distances, were seen bright and mountainous, or rather flame-like, protuberances. The greater number of these, according to the accounts collected from the different observers, seemed to be in contact with the moon's limb, and were broader at the bottom than at the top, but one at least was seen distinctly separated, and suspended, as it were, in the atmosphere surrounding the sun. In one direction, many observers agree in noting a large red *sierra*, or long mountainous-looking range, with an irregular tooth-like edge. Another protuberance had the shape of a sickle with the top broken off, and with another irregular-looking mass very near it, but neither in contact with it nor the moon's edge. Such prominences as were near that part of the sun's limb towards which the moon was moving, were observed to decrease in height (the moon passing over them), while those on the opposite part of the limb increased in height. It is evident, therefore, that these wonderful phenomena belong to the sun and not to the moon, and it is probable that they are of the nature of illuminated clouds suspended in the atmosphere surrounding the luminous envelope. There is little doubt also that the luminous ring or corona belongs to the sun and not to the moon, though its boundaries are so vague (the light becoming gradually weaker as the rays recede from the centre) that it was impossible in general

to determine whether it was concentric with the sun or with the moon.* (See Vignette.)

170. The last phenomenon which we have to notice in connexion with the sun, is the *zodiacal light*. If we look at the western part of the sky on a fine spring evening after the setting of the sun, we observe a brightness which we feel at first tempted to attribute to the still lingering twilight, except that it is considerably more intense; but, if we continue to watch this bright part of the sky till the sun has gone down so far as to prevent any of his rays from being reflected or refracted by our atmosphere, or till all effect of twilight has disappeared, the light still remains, and its intensity may be judged of by comparing the western with the eastern sky. The one exhibits a fine straw-coloured glow of light, fading away and becoming lost about forty or fifty degrees from the horizon; the other exhibits the dark blue which is characteristic of the sky during a fine winter night. This light is broadest at the horizon, and tapers away gradually till it is finally lost in a point nearly in the direction of the ecliptic. Indeed, if it be accurately observed, it will be found to have nearly the ecliptic for the axis of its lenticular or conical figure, and will be seen, if Venus be in the sky, to extend beyond her. It is very faint in this climate, and it is very difficult to determine the exact limit at which it terminates; and though it evidently attends the sun, being always observed in the direction of the ecliptic or sun's path, preceding him in the morning, before sunrise, in the autumn months, and lingering behind him, and lengthening our twilight, in the spring, after he has set in the evening; yet philosophers have not been able to form any satisfactory theory concerning it. It is one of those wonders of the heavens, which, for the present, we must be content with admiring, in the hope that the progress of science will, at some future time, give us some information respecting its cause and formation, and give us additional means for adoring in His works the great Architect of the universe, who by His word made and placed in this wonderful order all these orbs of heaven, whose motions it is the highest praise of man's intellect partially to understand and explain.

171. From the action of the rays of this great body, the sun, arise all the motions that are discovered on the surface of our planet. To the disturbances in our atmosphere,

* See *Notice of the Royal Astronomical Society for January 9*, 1852, for the collected accounts of observations.

produced by the unequal action of the heat coming from him, we owe the breeze that refreshes us in summer, and the storm that drives away infection from our dwellings. By his agency water retains its liquid properties, and the rains which he has forced from the clouds descend to the sea, again to be raised in vapour for the renewal of the same fertilising process. His action forces the sap into the vessels of trees, shrubs, and vegetables, and covers the earth with verdure and plenty. He is turned to us obliquely in winter, and his action is partially suspended: the rivers then become congealed and the moisture of the ground is converted into ice; snow covers our fields, and protects while it covers them. The action of the teeming earth is for a time usefully suspended to gather fresh strength for the coming season, when the sun's increasing warmth shall renew the energies of vegetation, and the trees shall again put forth their buds and their leaves: Nature then bursts forth again, as if refreshed with her long wintry sleep, and the marvellous enonomy of agricultural processes is renewed for the benefit of mankind.

CHAPTER V.

THE MOON.

172. Next to the Sun, the most interesting body in the solar system is the Moon. Independently of its use in regulating our minor divisions of time—that is, by the divisions of the year into months—it is of incalculable use in nautical science, and its motions afford the only means for the accurate determination of the place of a ship on the ocean, or for measuring the terrestrial longitude reckoned from a fixed point of departure. The proximity of this planet also is another interesting feature in all our discussions concerning her. By means of observations made with good telescopes, her surface has been mapped with almost the same accuracy as that of the earth; every prominent or conspicuous point is laid down in the lunar charts with the same fidelity with which our own mountains, and seas, and rivers have been depicted; the heights of her mountains have been measured,

and the physical peculiarities of her surface are now tolerably well known.

173. On all these accounts the knowledge of the motions of our satellite becomes an object not only of very great interest but of absolute necessity. The theoretical astronomer speculates upon the peculiarities of her orbit for the advancement of abstract science, but the merchant and the sailor reap the reward of his labours in the safety of precious cargoes and still more precious lives; and, without her aid, our ships, instead of fearlessly traversing the ocean from pole to pole, would probably even now be incapable of performing long voyages, but would content themselves with exchanging commodities and intelligence between well-known and neighbouring shores.

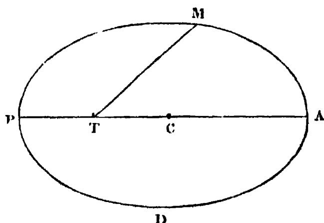
174. We arrive at a knowledge of the chief elements of the lunar orbit in the same manner as for that of the sun, that is, by observations carried on from day to day. The most careless observer cannot but take notice that her place in the heavens varies rapidly from one day to another; that she has in fact a daily motion from west to east, or in opposition to her diurnal rotation, of about three quarters of an hour—that is, that her time of arriving at the meridian is on the average retarded daily by that quantity. By simply mapping out her path in the heavens on a globe by means of the known stars that happen to be near her, from night to night, it would be easy to see that the apparent path in the heavens is nearly a great circle of the sphere, and that this great circle makes a small angle with the ecliptic. If, indeed, we were to apply the corrections to her observed right ascensions and declinations which are due to refraction, parallax, &c., according to the principles explained in Chapter III., we should find that her path was accurately enough for one revolution a great circle inclined at an angle of very nearly 28° to the equator, or of 5°, as aforesaid, to the ecliptic.

175. By observations also made near the time of her crossing the ecliptic, where her latitude is nothing, the calculated longitudes and latitudes will afford means, by simple proportion, of determining the longitude of the point where she crosses the ecliptic, that is the longitude of her *node*. If also observations be continued through several revolutions, and the longitude of the node be thus determined, it will be found that this point is not fixed, but has a *backward* motion along the ecliptic, similar in its character to

the *precession* of the equinoxes, but incomparably more rapid. In fact, observations continued throughout the space of several years show that the *nodes* regress through the whole circle, or come back to the same point of the ecliptic, or perform a revolution, in the space of rather less than nineteen years.

176. Again, when we inquire into the nature of the curve which she describes round the earth in the plane of her orbit, observations of a similar character to those detailed in our chapter on the solar motion will show us that her orbit is on the whole elliptical, thus obeying Kepler's First Law; and a comparison of her velocity (measured by her daily change of longitude), when at her greatest and least distances (determined by her apparent diameters measured at those times), will show us that Kepler's Second Law of elliptic motion is also obeyed: viz., *that the areas swept out by the radii vectores are proportional to the times.*

177. A comparison also of the greatest and least diameters will show that the excentricity of the ellipse is about 0·0635. Thus, if P and A, be the *perigee* and *apogee* of the orbit round the earth T, (that is the points of least and greatest distance); C the centre of the ellipse—then T C is the excentricity, supposing A C the mean distance to be unity. Let D_1 and D_2 be the observed diameters at P and at A:



$$\text{Then, } \frac{PT}{AT} = \frac{D_2}{D_1}; \text{ or, } \frac{1-e}{1+e} = \frac{D_2}{D_1}; \text{ whence, } e = \frac{D_1 - D_2}{D_1 + D_2}.$$

If $D_1 = 33' \, 30''$ and $D_2 = 29' \, 30''$, which are nearly the limiting observed values of the diameter, $e = \frac{4}{63} = 0·0635$, agreeing with the value given above.

178. Now, if the longitudes of the perigee and apogee of the orbit be observed, by means of the knowledge gained by the variations of the diameter, or, which is better still, by the variations of her velocity or daily increase of longitude; it will be found (as was the case in the solar orbit), that these points are not fixed, but have a direct motion, that is, in the direction corresponding to increase of longitude, so

rapid as to carry them round the whole circle in about nine years. This revolution of the perigee, like that of the node, is not uniform, that is, it is sometimes quicker and sometimes slower than its mean value, and sometimes even regressive; and it has a *secular* variation, that is, its mean motion, derived by observations made at two distant epochs, would not agree with the mean motion determined by observations made at other two distant epochs. But at the beginning of the present century, the motion was such that the time of a tropical revolution of the perigee was 3231·475 days.

179. There is still one more peculiarity of the motion of the moon to be taken notice of, viz., the *acceleration of the mean motion.* For example, if the moon's place be computed without regard to the acceleration for the epoch of an ancient eclipse such as the Babylonian eclipses transmitted to us by Ptolemy, which were made several centuries before the Christian era, we find the longitude thus computed to differ by nearly a degree and a half from the longitude computed by the conditions of the eclipse, that is, by the known relative positions of the sun and moon. The moon's place, computed by the eclipse, is in fact in all such cases greater than that computed from the tables; that is, the tables have thrown the moon's place too far back, or too great a mean motion has been allowed. This *acceleration of the mean motion of the moon* was first discovered as a fact of observation by Halley, and its physical cause was afterwards deduced from the theory of gravitation, by Laplace, and was shown to depend on the secular diminution of the excentricity of the earth's orbit. It is a very remarkable example of that class of equations or corrections of long period, known by the name of *secular equations.* Though it has existed since the earliest ages of astronomical observation, still it is *periodical* and not *permanent;* that is, after a great number of ages it will be reduced to nothing, and after that the motion, instead of being *accelerated* will be *retarded,* through an equally long period. If the fact were otherwise, the moon must, in however remote a period, be drawn in towards the earth with a still accelerated motion, and would at length be attracted to and fall on its surface. The motion of the node and that of the perigee are also both subject to *secular equations.*

180. The above remarks will serve to show how the motion of the plane of the lunar orbit and the motion of the moon in that plane have been observed. By refined processes depending on such observations, the value of each of the fore-

going elements has been obtained with very great accuracy. Thus, for the beginning of the present century (1801), the longitude of the ascending node was 13° 53′ 22″, and its time of revolution round the ecliptic was 6798 days, or about eighteen years and six months; while the time of a *synodic revolution*, that is, the interval between two successive meetings with the sun, was 346·62 days, the node having gone backwards on the ecliptic to meet the sun through an arc of 19° 20′, nearly. The mean inclination of the lunar orbit to the ecliptic is about 5° 8′ 48″, and it varies from about 5° to 5° 17′. The direct motion of the perigee for the beginning of the century was, as has been said before, such as to complete a *tropical motion* (that is, setting out from the equinox and returning to it again), in 3231·475 days.

181. The time of revolution of the moon in her orbit may, as in the case of the sun, be distinguished into *tropical*, *sidereal*, and *anomalistic*. The time of a *tropical* revolution is the interval between the departure of the moon from the equinox till its return to it. The time of a *sidereal* revolution is the interval between the departure from and return to the same point of the heavens. And an *anomalistic* revolution comprises the time between the departure from perigee or apogee till the return to it. In addition, for the moon we have a *synodic* revolution, that is the interval between the departure from and return to conjunction with the sun, which depends on the relative motions of the two bodies. We have not space in so brief a treatise to repeat the reasoning by which these values are derived from the tropical period by means of the elements given above. It is sufficient here to give the values, leaving the student to exercise himself by deducing them one from the other.

182. By very exact observations of the mean motion of the moon with regard to the equinox made at the interval of a century, it is found that the value is 13° 10′ 35″; hence the time of describing 360°, or that of a tropical revolution, is $27^{d}\ 7^{h}\ 43^{m}\ 4^{s}{\cdot}7$.

183. From this can be deduced, by means of the values of the sun's mean motion, the annual precession, and the motions of the lunar node and apogee, the following values:—

		d.	h.	m.	s.
Time of Synodic revolution	=	29	12	44	2·8
„ Sidereal revolution	=	27	7	43	11·6
„ Anomalistic revolution	=	27	13	18	37·4
„ Draconic revolution, or revolution with relation to the node	=	27	5	5	35·6

184. Thus far we have considered the moon to move in an elliptic orbit which may be considered to be described in a plane having a *direct* motion of revolution round an axis passing through the earth's centre, while its point of intersection with the ecliptic retrogrades, and its inclination to that plane has periodical changes. But this supposition is only approximately true. The moon does not move truly in such an ellipse, and it is only for convenience, or for the greater facility of calculation of her true place, that she is supposed to do so. If, for example, we were to take the elliptic elements for a given epoch, and, by the known value of the excentricity, construct, as in the case of the solar orbit, a table of the values of the *equation of the centre*, this quantity added to her mean anomaly would not give her true place in the heavens for any given time, after making every allowance for the retrogradation of the node, the variation of the inclination of the orbit, and the progression of the apogee. In fact we should find, by a very simple comparison of calculated and observed places, that the calculated place was sometimes considerably in advance of, and sometimes behind the observed place.

185. The first and largest of the inequalities thus discoverable by observation is called the *evection*, and was discovered by Ptolemy, having escaped the observation of his celebrated predecessor, Hipparchus. In fact Hipparchus, whose observations of the moon were made only at conjunction and opposition, had no opportunity of recognising the evection, because at those times it became confounded or mixed up with the equation of the centre, and simply showed too small a value of this equation, or of the excentricity of the orbit. Ptolemy, however, who observed the moon at *quadratures*, that is, at 90° distance from the *syzygies* or conjunctions and oppositions, discovered the true law of the inequality, which has for its argument twice the difference of the longitudes of the moon and sun, *minus* the moon's mean anomaly, and varies as the sine of this quantity. Its greatest value, or the *coefficient* of the preceding argument, is about 1° 20′ 29″·5.

186. The physical cause of the *evection*, and indeed of almost all the inequalities of the moon, except the equation of the centre, is the disturbance produced by the sun's action on the relative orbit of the earth and moon, and of this we will attempt a brief explanation. It admits of easy proof from observation that this inequality depends on the position of

the line of apsides, that is, of the major axis of the lunar orbit; for, if this line be *in syzygy*, that is, coincident with the line joining the sun and earth, the moon, after moving from apogee through a quarter of her orbit, that is, to quadratures, will be found *behind* the place computed from the equation of the centre, by more than a degree, but if the line of apsides be in quadrature, her true place, some days after quitting the apogee, will be found to be *before* the computed place by nearly the same quantity. Imagine now the line of apsides to be in syzygy; then, since the moon moving from apogee has been found behind her computed place, too small a correction has been subtracted for the equation of the centre, or the equation of the centre is apparently increased, denoting an increase of excentricity. If, on the contrary, the line of apsides be in quadrature, the equation of the centre, and the excentricity, would appear to be diminished. Hence the observed effect of the evection is to increase the excentricity of the orbit when the apsides are in syzygy, and to diminish it when they are in quadrature.

187. Let us now consider whether the action of the sun in disturbing the lunar orbit, will give an adequate explanation of this and other observed inequalities. For simplicity, we will first suppose the undisturbed orbit to be circular, and trace out some of the consequences of the disturbing force on this supposition: evection depends upon the excentricity of the orbit, and, though the largest of the inequalities, and coming first as a fact of observation, does not admit of so easy a popular explanation. It must be remembered, that the effect of the sun in disturbing the moon depends on the *difference* of his actions upon the earth and the moon. The sun tends to draw both bodies towards his centre, and if they were at equal distances from him, and moving in parallel directions with equal velocities, it would draw them through equal spaces in equal times, and their relative orbits would not be disturbed. But none of these conditions generally hold; the direction of the motion of the moon, her velocity, and her distance from the sun, are perpetually varying during her revolution round the earth, while, for one revolution, these quantities may be considered for the earth pretty nearly constant; hence disturbances arise in the orbit, and it is plain that the points of greatest and least distance from the sun, that is the points of syzygy, will be those at which she is particularly affected,

since at conjunction she is nearest to him, and at opposition farthest from him. If she be nearest to him, or at conjunction, then she is pulled towards him more than the earth is, and, her gravitation to the earth being lessened, the curvature of her orbit is lessened; if, on the contrary, she be farthest from him, she is pulled less towards him than the earth, and the curvature of her orbit is also lessened. In both these cases the disturbing force is directed *from* the earth in the direction of the radius vector. But at any intermediate point, the disturbance in the direction of the radius vector will not be the only one, since the sun does not draw the moon in the direction of that line. In all cases, however, (neglecting the inclination of the orbit to the ecliptic,) the disturbance can be always resolved, by the ordinary laws of mechanics, into two, one of which is in the direction of the radius vector, and the other at right angles to it, or approximately in the direction of a tangent to her motion, her orbit being nearly circular. These forces are called the *radial* and the *tangential* disturbing forces; the first increasing or diminishing the moon's gravitation to the earth, the latter increasing or diminishing her velocity in her orbit.

188. If we were again to take the case for which the moon is in quadratures, and her distance from the sun nearly equal to the earth's distance from the sun, then the sun will pull them equally, but not in the same direction. Thus let S, T, and M be the positions of the sun, earth, and moon (near quadrature), when $ST = SM$. Then if, in a given time, the sun pull the earth and moon in equal times through the spaces Mm and Tt, these spaces will ultimately represent the forces exerted upon each. Now Mm may be resolved into Mr, in the direction of TM and mr, parallel to Tt. Hence, in this case, Tt and mr ($= Mm$), being equal, the effective disturbing force is represented by Mr, in the direction of the radius vector. Hence at quadratures, as well as at syzygies, the disturbing force is in the direction of the radius vector, but at quadratures it is directed *towards* the earth, and is much less than at syzygies. At any inter-

mediate part of the orbit the force will be partly *radial* and partly *tangential*, and the effects of these respectively in increasing or diminishing the gravitation to the earth, and in increasing or diminishing the velocity in the orbit, can be traced out for any assigned position of the orbit by similar resolution of the disturbing force. We may see, however, immediately, that since at M, the radial force is directed towards T, and at A it is directed from T, there is some intermediate point where it vanishes, and where the whole disturbing force is tangential, and similarly for the other three quadrants. With respect also to the tangential force, it may be shown that it accelerates the motion of the body from quadrature to syzygy, but retards it from syzygy to quadrature.

The above considerations will tend to show the general effects of the sun's disturbing force upon the orbit which the moon would describe in a revolution, if it be supposed circular; and in particular, it will serve to show the nature of one of the lunar inequalities, called the *variation*, which depends upon the oval shape of the orbit, and on the action of the tangential force, and causes the angular motion of the moon to be greater than the mean at syzygies, and less than the mean at quadratures, and therefore causes the moon to be before her mean place from syzygy to quadrature, and behind it from quadrature to syzygy.

189. Thus far we have proceeded on the supposition that the moon's undisturbed orbit is circular, or has no excentricity. The introduction of the consideration of the excentricity does not greatly concern the moon's variation, but it is immediately concerned in the explanation of the greatest of the *inequalities*, viz., the *evection*, which we proposed to explain. The reader will, for the present, take for granted, without farther explanation than that offered in the discussion of the solar orbit, that by the action of the disturbing forces, the major axis of the excentric orbit, that is, the line of apsides, is made to revolve with a direct motion; that is, that the perigee progresses, with, on the whole, a rapid yet very irregular motion. As the relative positions of the earth and moon are very much affected by this circumstance, it is evident that the effect of the disturbing forces in producing inequalities, will also be much affected by the position of the line of apsides. The principal effects may be thus described, though we must refer the reader to "Airy's Gravitation" for a complete explanation. If the lines of

apsides be either in syzygy or in quadrature, though, during a revolution of the moon the excentricity is alternately increased and diminished, yet, on the whole, the excentricity is neither increased nor diminished. If, however, the line of apsides be inclined in such a manner that the moon passes the apsides (perigee or apogee) before passing the line of syzygies, the excentricity is diminished at every revolution of the moon. If, finally, the position of the line of apsides be transverse to the former, so that the moon passes it after passing the line of syzygies, the excentricity is increased in every revolution of the moon. As, then, the earth in its real orbit, or the sun in its apparent orbit, is carried round in its annual circuit, the line joining them is brought into every position with respect to the line of apsides of the lunar orbit, and the excentricity will be either increasing or diminishing with very variable velocities. If the apsides are in syzygy, the excentricity is at its greatest value, and stationary; if, on the contrary, the apsides be in quadratures, the excentricity has its least value, and is also stationary; finally, it diminishes while the sun is moving from the syzygy position of the apse till it is at right angles to the line of apsides; and it increases from this position till the apse is in syzygy again. Hence, since the equation of the centre depends on the excentricity, there will be an inequality introduced depending on the moon's mean anomaly, as well as on her distance from syzygy. This is the evection, the largest of the lunar inequalities, and it has for its argument as has been stated, twice the difference of the longitude of the moon and sun *minus* the moon's mean anomaly. We have no space to go through the whole of the reasoning for the explanation of all the inequalities, and, indeed, the reader will find them all admirably discussed in "Airy's Gravitation," forming one of the articles in the "Penny Cyclopædia," but reprinted in a separate volume. We will simply mention in few words the conclusions arrived at with regard to the three great inequalities, the *evection*, the *variation*, and the *annual equation*. The former, that is, the *evection*, is dependent upon the position of the line of apsides, and is derived from two effects of the sun's disturbing action, viz. the irregularity of the motion of the perigee, and the periodical alteration of the excentricity of the orbit: the *variation* arises from the action of the tangential force, which forces the orbit, supposed circular, into an oval shape, and alternately diminishes and increases the velocity; it depends, therefore, upon the

distance of the moon from the syzygies: finally, the *annual equation* arises from the excentricity of the earth's orbit, which causes the sun's distance to vary, and the disturbing force in consequence to vary, at different periods of the year; its period is therefore annual, and hence its name.

We have shown the form in which the *evection* is introduced into calculation.

The *variation* amounts at its maximum to about 35′ 41″·6, and has for its argument the sine of twice the difference of longitude of the sun and moon. It was discovered by Tycho Brahe about the year 1590.

The *annual equation* amounts at its maximum to 11′ 11″·97, and has for argument the sine of the sun's mean anomaly. It was also discovered by Tycho Brahe about the year 1590.

190. For computation of the moon's true longitude, we have then, neglecting minor inequalities,—

True longitude = mean longitude
+ Equation of centre + Evection
+ Variation + Annual equation,

which will become, if we put E, to denote the moon's mean longitude at a given epoch, M her mean anomaly, reckoned from apogee, and ☽ and ☉ the moon's longitude and the sun's—

$$\begin{aligned} \text{True longitude} = \text{E} &- 6^\circ\ 17'\ 54''\cdot40 \times \sin.\ \text{M}. \\ &- 1^\circ\ 20'\ 29''\cdot5 \times \sin.\ [2\,(☽ - ☉) - \text{M}] \\ &+ 35'\ 41''\cdot6 \times \sin.\ 2\,(☽ - ☉) \\ &+ 11'\ 11''\cdot97 \times \sin.\ ☉ \end{aligned}$$

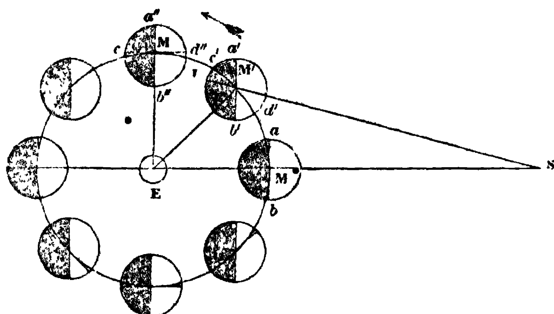
In the preceding discussion of the orbit described by the moon round the earth and its irregularities, we have only endeavoured to familiarise the student with some of the leading features of the lunar theory, as preparatory to the complete and philosophical popular explanation which he should seek for in the reading of "Airy's Gravitation." In that treatise not only the perturbations of the moon, but the planetary perturbations generally, are fully discussed, by means of reasonings deduced from the simplest principles of mechanics, and without the use of a single algebraical expression. It is, therefoie, capable of being studied by any one possessed of ordinary abilities, and should be neglected by no student who aims at a clear conception

of those complicated laws of planetary movement arising from their mutual perturbations, which he will afterwards have to develop to their remotest consequences by the most refined and difficult processes of analysis.

191. We will now proceed to a much easier and more obvious subject, viz. the *phases* of the moon. In watching the moon through a lunation, we not only observe a rapid orbital motion from west to east amongst the stars, but (evidently in connexion with this orbital motion) a change of figure and magnitude of the illuminated portion of the disk. For example, after missing her light for several evenings, we observe her at a short distance following the sun in the form of a thin crescent, with its convexity turned towards him. From evening to evening as she separates from the sun by her relative easterly motion, the crescent increases in magnitude, till the line joining the horns is exactly filled with light; she is then distant from the sun by about six hours of Right Ascension, and is said to be *dichotomised*. She then becomes *gibbous*, or is more than half illuminated, and after separating from the sun by twelve hours of Right Ascension, has the whole disk illuminated, or is at the full. After this, she wanes, or the illuminated portion of the disk (still turned towards the sun, and therefore, with regard to the east and west points of the horizon, illuminated on the side opposite to that of the increasing moon) becomes less and less, till we see her again early in the morning as a thin crescent, with its convex edge turned eastwards, and we then finally lose sight of her till after her conjunction with the sun, when the same cycle of changes is renewed.

Now these changes, or *phases*, are easily explained, on the supposition of the moon being an opaque body, made visible to us by light reflected from the sun. On this supposition (which admits of no doubt) the half of the moon's disk turned towards the sun will be illuminated, while only that half turned towards the earth will be visible. If then the moon be in conjunction with the sun, or between the sun and the earth, her dark side is on the whole turned towards us; but being generally either above or below the sun, a small portion of her upper or lower limb is still visible while the crescent changes from the eastern to the western limb. Again, when she is in such a position with respect to the earth and sun, that her disk is *dichotomised*, or half illuminated, the moon is in such a position, that the line joining the

moon and earth is perpendicular to that joining the moon and sun, the angle formed by these lines having been before acute; and, after this, the obtuseness of this angle still increases till the moon is in opposition, that is, till the earth is nearly in a direct line between her and the sun, when, this angle becoming equal to two right angles, nearly the whole disk is illuminated.



192. The accompanying figure, in which S represents the sun, E the earth, and M the moon in its orbit, will exemplify the changes above described. We here suppose the moon's orbit to coincide with the ecliptic, which is sufficiently correct for the general explanation of the phenomenon. If also, on the figures of the lunar disk we draw through their centres lines perpendicular to the line S E, the semi-circles thus cut off opposite to the sun represent with sufficient exactness the orthographic projections of half the illuminated hemispheres of the moon, since the whole circle of the lunar orbit subtends at the sun an angle not amounting to a degree. Also the lines $c'\ d'$, $c''\ d''$, &c., at right angles to the radii vectores, E M, E M$'$, cut off semi-circles opposite to the earth representing the projections of the hemispheres visible from the earth. The inclination of the lines $c'\ d'$, $c''\ d''$, therefore, in any position of the moon, to $a'\ b'$, $a''\ b''$, will measure the portion of the illuminated surface, visible from the earth, and generally if we suppose M$'$ d' to set out from its initial position M b, where the moon is in conjunction, this angle bears the same proportion to two right angles, that the illuminated disk does to the whole disk of the moon. But it is evident from what has been said (the sun's distance being so great) that the lines S M$'$, E M$'$ are

sensibly perpendicular in all cases to b' M′, d' M′, and therefore the illuminated portion is measured by the exterior angle L M′ E of the triangle E M′ S, or, as it is called, by the exterior angle of elongation. Now on account of the distance of the moon, we see all the parts of her surface orthographically projected on the plane passing through her centre, perpendicular to the line joining her with the earth. Hence it is the *versed sine* of this angle that measures the illuminated surface which we actually see, and this surface will in any case be found equal to moon's surface × versed sine of exterior angle of elongation.

193. If we take the case for which the angle E M′ S is a right angle, or for which the moon is half full or dichotomised, then since the angular distance M′ E S of the sun and moon can be measured, it is plain that the proportion of the distances E M′ and E S, that is, of the distances of the moon and the sun, can be determined. It was in this way that the astronomer Aristarchus, of Samos, in the third century before the Christian era, formed a tolerably correct estimate of the relative distances of the sun and moon.

194. There is still one phenomenon more, which we must give some account of, viz. the *libration* of the moon. In all probability the motion of the moon round her axis of rotation is uniform, and this motion she performs round an axis inclined at an angle of $1\frac{1}{2}°$ to the ecliptic, in the same time as that of her revolution in her orbit. This is found to be the case by observations of some of the conspicuous spots on her surface, and hence arises the circumstance that, on the whole, we always see the same disk of the moon. But since the motion in her orbit is sensibly unequal, being sometimes faster and sometimes slower than the mean, a little more of the eastern and of the western limb is in the course of a revolution brought into view than would otherwise be the case, and this is called the *libration in longitude.*

195. The *libration in latitude* is caused by the axis of rotation of the moon not being exactly perpendicular to the ecliptic. On this account, in the course of a revolution in the orbit, her northern and her southern poles are alternately presented to us, and a little more of her northern and her southern surface is visible to us in the neighbourhood of the poles than would otherwise be the case.

196. There is a third kind of libration called the *diurnal* libration, which arises from the observer's position on the

earth's *surface*, instead of at the *centre*. The moon turns constantly the same hemisphere, not towards a point on the surface, but towards the centre of the earth. Now, on account of the small distance of the moon, the line joining her centre and the earth's centre, changes its direction with regard to that joining her centre and the observer's position on the surface as she rises above the horizon. If she were exactly in the zenith, these lines would exactly coincide, but, in any other position, we see more of the upper part of the surface near the limb than we should see from the centre of the earth, and less of the neighbourhood of the lower limb, and by a variable quantity depending on her height and distance, that is, on her parallax.

197. We will finally make a few remarks on the physical peculiarities of the surface of the moon, which, on account of her proximity, are better known to us than those of any other body. By the use of good telescopes we see her surface broken up into irregular patches of light and shade, which evidently indicate inequalities of considerable magnitude. When the sun's light falls most obliquely on the surface presented to us, near the conjunctions for example, we find the boundary of light and darkness not to consist of a regular or well defined line, but of a series of jagged luminous points, some of which are at a considerable distance from the generally illuminated curve. These are plainly the tops of mountains that catch the first rays of the sun, while the intervening valleys are left in darkness. The heights of some of these which have been deduced by ingenious mathematical processes from micrometrical observations made of them, are very great, in fact, considerably higher than any on the surface of the earth. These mountains, which are extremely numerous, present every characteristic of volcanic formation. They are almost universally of a circular shape, and the larger ones have a hollow within their circular boundary, terminating in a flat bottom, and in some cases having a steep conical hill in the centre. Our readers will, doubtless, be all familiar with the excellent charts of the surface which, during the last summer, were exhibited in the Crystal Palace. Mr. Nasmyth, the maker of them, who is equally eminent for his engineering skill and his mechanical ingenuity, has, with telescopes of his own construction, given much attention to observations of the moon, and presented to the Astronomical Society, some years since, a model of about 200 miles of the surface, admirably formed,

and exhibiting the relative elevations and general character of the most conspicuous of these volcanoes. A visit to this model will be well repaid by the accurate knowledge which will be gained; but an ordinarily good telescope will show very satisfactorily some of the most remarkable spots. As viewed by Lord Rosse's gigantic reflecting telescope, the flat bottom of one crater is seen to be strewed with blocks or large stones, while the exterior of another is "hatched over" with gullies, radiating towards its centre.*

198. The moon has never been discovered to have on her surface the slightest trace of seas or water of any kind, yet there exist large tracts of apparently alluvial formation, indicating that water must have existed at some previous time. Neither has she any clouds or vapours or any other decisive indications of an atmosphere. This is proved chiefly by the observations of the occultations of stars and of solar eclipses, for the interval of time between the disappearance and reappearance of a star when occulted by the moon, and the time taken in transiting the solar disk during an eclipse, is evidently not affected by such an error as would be produced by refraction of the rays in passing near the moon's edge through an atmosphere of any sensible density. Nevertheless philosophers are not even now all agreed on the total absence of a lunar atmosphere; it is, however, quite clear that if any exist it must be of extreme tenuity, and it is not discoverable by any observations which we can make from the surface of the earth.

199. On account of the slow rotation of the moon, making, that is, a complete revolution in twenty-nine days, the surface is alternately exposed to the heat of the sun unmitigated by clouds or vapours for half that period, and during the other half to the severest intensity of frost. It is evident from these remarkable features of climate, and from the want of an atmosphere, that no inhabitants whose physical organs are at all similar to our own would be able to exist at her surface. The force of gravity there is also small (only one-seventh) compared with that at the earth's surface, as determined by her mass deduced from very elaborate mathematical discussions of her effect in producing *lunar nutation* by acting on the protuberant matter at the earth's equator; but all the circumstances combined seem to show that nothing like animal life exists there. She gives light to us, and fills our hearts with gratitude to the Giver of all

* Outlines of Astronomy, p. 259.

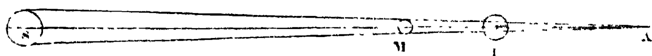
good, for his mercy in thus providing for our comfort and our safety, while a nearer inspection saddens the imaginative mind by presenting the image of a wrecked or a burnt-up planet, a monument of vengeance perhaps in by-gone ages on a guilty world, and awaiting the Almighty's fiat to become at some future period equally distant, again a dwelling-place for other organised beings. We cannot close this description of the surface of the moon, without bringing before the reader's attention the well-known exclamation of the pious David, which will be repeated with fervour by all those who have studied deeply these wonderful bodies, whose motions and properties we have been considering:—"When I consider the heavens, even the works of thy fingers: the moon and the stars, which thou hast ordained. What is man, that thou art mindful of him: and the son of man, that thou visitest him?"

200. In connexion with the motions of the sun and moon, it will be expected that we give some explanation of the eclipses of those bodies. This we shall do in few words, referring the reader for fuller information to Sir J. Herschel's "Outlines of Astronomy," or other popular works, since our necessary limitation of space enables us to give only an outline of the leading features and principles of astronomy, without dwelling much on the details of the *casual* phenomena resulting from the motions of the heavenly bodies.

201. Since the inclination of the lunar orbit to the ecliptic is small (only 5°), and since there are more than twelve conjunctions and twelve oppositions of the moon with the sun in the course of every year, it follows that there is a very great probability of the sun, the earth, and the moon, being so nearly in a straight line at some of these times of conjunction or opposition as to produce an eclipse of the sun or the moon. If this should take place when the moon is in conjunction with the sun (that is, at new moon), it is plain that she will pass directly between us and that luminary, and prevent either the whole or part of his light from reaching us, or there will be an eclipse of the sun. But since by parallax she is depressed on the visible sphere of the heavens by a large quantity depending upon the geographical latitude, while the place of the sun, on account of his great distance, is in a very trifling degree affected; it is plain that an eclipse of the sun may take place at one point of the earth's surface, while there is no eclipse at all

at another, or it may be total or annular at one place, and only partial at another. Thus the recent great eclipse of 1851 was total for parts of Sweden, Norway, Prussia, and Poland, while at Greenwich it was only partial. Now, for finding the circumstances of a solar eclipse, that is, for determining whether it will be *total* or *annular*, and for finding those parts of the earth's surface at which it will be total or annular, and at which it will be only partial; it is necessary to remark, that the cone which would envelop the sun and moon, at the time of a solar eclipse, that is, which would be formed by a series of tangents to their surfaces, has its apex situated very nearly at the distance of the earth from the sun, being carried a little farther off, or a little nearer, accordingly as the moon happens to be nearer to, or farther from, the earth, or near perigee or apogee. If the moon be in or near to perigee, the apex of the cone will lie farther from the sun than the earth is, as in figure 1.,

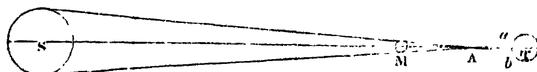
Fig. 1.



where S, M, and T represent the sun, moon, and earth; and the part of the earth's surface intersected by the cone during the progress of the moon across the sun's disk, gives the geographical boundaries of the eclipse, within which a total eclipse may take place, and near the centre of which it *must* take place.

202. If, however, at the time of the eclipse the moon be near *apogee*, the apex of the cone will lie between the sun and the earth as in fig. 2., and the sheet of the cone produced will meet the earth's surface, as at *a*, *b*. Hence a

Fig. 2.

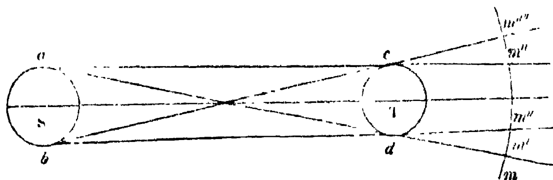


person at *a* will see the upper part of the sun, but the moon's lower limb will appear to graze the sun's lower limb; while an observer at *b* will see the lower part of the sun while the moon's upper limb appears to graze that of the sun. Between *a* and *b* an observer will see a little of the upper and lower limbs, or the eclipse will be annular.

203. Thus far we have spoken of annular and total solar eclipses, which occur when the moon is very near the node of her orbit at the time of conjunction. If she be at a greater distance from the node, a partial eclipse will take place at some parts of the earth's surface, while none at all may take place at others. The sun may also at the time of an eclipse be beneath the horizon, or it may occur during the night, and will, of course, to such places be invisible.

204. If the moon be near her node at opposition (or full moon), a *lunar eclipse* may take place by the interposition of the earth preventing the light of the sun from reaching her. Hence the cone used in illustration must be supposed to envelop the sun and earth, and the position of the moon with regard to its axis in passing across the shadow thus thrown upon her will determine whether the eclipse is total or partial. If she be very near her node at the time of opposition, a total eclipse will take place equally to every part of the earth's surface at which she is above the horizon; but if she be at some distance from the node, a partial eclipse or none at all will take place.

Fig. 3.



If in fig. 3, we draw tangents on the same and opposite sides of the earth's surface (T) from the sun S, viz. *a c*, *a d*, and *b d*, *b c*; it is plain that betweeen *a c*, and *b d*, no light of the sun can extend; but that between *a c* and *b c* produced, as also between *b d* produced and *a d* produced, part of the light of the sun can reach, and a *penumbra* will be formed during the progress of the moon from *m* to *m''''* between *m'* and *m''*, and again between *m'''* and *m''''*, when her surface will be only obscured, but that from *m''* to *m'''*, while she is in the *umbra*, or real shadow, the whole or part of her surface will be invisible.

CHAPTER VI.

THE PLANETS.

205. In discussing the motions of the planets, we shall assume that the reader is sufficiently familiarised with the reasonings by which it is proved that the sun is the comparatively immoveable centre of the system with which we are connected, and that the earth and all the moveable bodies called planets, move round it in elliptical orbits, whose excentricities are small, and whose planes generally (that is, for the large planets) are inclined at small angles to the ecliptic. It is sufficient also barely to intimate here, that Kepler's laws are applicable to all of them, that is, that, in addition to the description of elliptical orbits, they describe round the sun equal areas in equal times, and that the squares of their periodic times are as the cubes of their mean distances from the sun.

206. They also suffer perturbations from their mutual action on each other according to the law of gravitation, in the manner which we have already tried to elucidate, by which the planes and the elements of their orbits are slowly changed, and their places in their orbits disturbed. Thus, with the exception of Venus, the apsides of their orbits *progress*, while their nodes generally *regress*, on the plane of the ecliptic. The inclinations also of the orbits to the ecliptic suffer change; but, with respect to all the elements, it must be borne in mind, that the changes are *secular* (that is, periodical, with an exceedingly long period), and not uniform or permanent; and that the changes, whether of increase or decrease, will, after a very long time, be compensated by others of an opposite character. Thus the excentricity of the earth's orbit is slowly diminishing at present by the action of the other planets, but it will, after a very long time, become increasing; the same is the case

with the obliquity of the ecliptic, which is diminishing at present, but will ultimately increase in the same manner.

207. In particular, this important result has been discovered by analysis, that the major axes of the orbits are neither subject to secular inequalities, nor do they increase or diminish indefinitely with the time. They are affected only by periodical inequalities, somewhat of the same nature though very much smaller than those which we have explained as affecting the moon, depending on the configuration of the bodies amongst each other, and the positions of their nodes, &c., and returning to the same values as often as the attracting and attracted bodies return to the same relative situation.

The chief difference in the lunar and planetary perturbations is, that in the former case they proceed from the action of the sun, incomparably the largest body in the system, while the perturbations of a planet arise from the actions of other planets, whose masses are insignificant compared with that immense body. Hence, the disturbances of the moon are very much greater than those of any planet, and the elements of her orbit are not only very *much* changed but very *rapidly* changed in the comparison. Thus the apse of the lunar orbit makes a complete revolution in about nine years, while that of the solar orbit, that is, of the earth's orbit, would take more than one hundred thousand years to make a complete revolution. Again, the moon is sometimes disturbed in longitude in her elliptic orbit by more than a degree, while the perturbations of a planet amount to only part of one minute. It thus happens that we are obliged to use the true place of the moon in calculating the disturbances, while, with regard to the planets, it is sufficiently accurate to take the position they would have occupied if undisturbed.

Without dwelling any longer on the difficult subject of the planetary perturbations, we will proceed to the more popular and generally interesting information which we propose to give, concerning the individual members of the solar system, confining ourselves to the most remarkable fact in the disturbances of each separate body which we have occasion to describe.

208. At the commencement of the present century, the number of the primary planets then known was only seven, including the earth, all of which, excepting the last, were known to the ancients. As a remarkable proof of the intel-

lectual activity of the present age, and particularly of the attention which has been devoted to Astronomy, the number known at present is twenty-three. Of these additional planets, twelve have been discovered since the year 1845, eleven of which lie between the orbits of Mars and Jupiter, and are of the nature of those bodies commonly called asteroids which were discovered at the beginning of the century; the remaining one is the planet Neptune, exterior to all the rest, whose discovery is one of the greatest intellectual triumphs of the present time.

209. Before the discovery of Neptune, the mean distances of the planets from the sun were observed to obey a very curious empirical law (called Bode's law, that astronomer having first noticed it) which may be thus expressed. Call the distance of Mercury, 4, then that of Venus is $4 + 3 = 7$; that of the Earth is $4 + (3 \times 2) = 10$; that of Mars, $= 4 + (3 \times 2 \times 2) = 16$; that of the small planets $= 4 + (3 \times 2 \times 2 \times 2) = 28$; and so on, the distance of Uranus being 196. But, for the next planet, the distance thus computed would be 388, which, the reader will perceive by inspection of the table which follows, is considerably too great. Now, in the calculations made by Mr. Adams and M. Le Verrier previously to the discovery of Neptune, it was absolutely necessary to assume arbitrary values for the mean distance of the supposed disturbing planet, and there was no clue whatever to guide these mathematicians in their assumption, except that afforded by Bode's law. It thus happened that several sets of calculations were found necessary, the assumed value being continually lessened, before the conditions of disturbance were satisfied, and even at the last the distance finally assumed proved to be too large. It is therefore an even question, whether the law was of any service in the calculations or not, but at all events it formed a kind of basis for the commencement of the work, and the errors of the assumptions were necessarily capable of correction by the processes employed.

210. Before proceeding farther we will give a table of the names, characters, and approximate mean distances of the planets now known, with the names of their discoverers and the dates of discovery.

TABLE OF THE NAMES, CHARACTERS, AND APPROXIMATE MEAN DISTANCES OF THE PLANETS NOW KNOWN.

Name of Planet.	Character.	Mean distance.	Discoverer.	Date of discovery.
MERCURY	☿	0·3871	..	Ancient.
VENUS	♀	0·7233	...	...
EARTH	♁	1·0000	...	...
MARS	♂	1·5237	...	...
FLORA.	⚘	2·2017	Hind.	1847, Oct. 18.
VICTORIA	[illegible]	2·3349	Hind.	1850, Sept. 13.
VESTA	⚶	2·3611	Olbers.	1807, Mar. 29.
IRIS	[illegible]	2·3806	Hind.	1847. Aug. 13.
METIS	[illegible]	2·3856	Graham.	1848, April 26.
HEBE	[illegible]	2·4255	Hencke.	1847, July 1.
PARTHENOPE . .	[illegible]	2·4255	De Gasparis.	1850, May 11.
EGERIA		2·5792	De Gasparis.	1850, Nov. 2.
ASTRÆA	[illegible]	2·5770	Hencke.	1845, Dec. 8.
IRENE		2·5842	Hind.	1851, May 19.
EUNOMIA	[illegible]	2·6176	De Gasparis.	1851, July 29.
JUNO	⚵	2·6708	Harding.	1804, Sept. 1.
CERES	⚳	2·7681	Piazzi.	1801, Jan. 1.
PALLAS	⚴	2·7729	Olbers.	1802, Mar. 28.
HYGEIA	[illegible]	3·1315	De Gasparis.	1849, April 12.
JUPITER.	♃	5·2028	...	Ancient.
SATURN	♄	9·5388	...	...
URANUS.	♅	19·1824	Sir W. Herschel.	1780.
NEPTUNE	♆	30·0368	Adams and Le Verrier.	1846, Sept. 23.

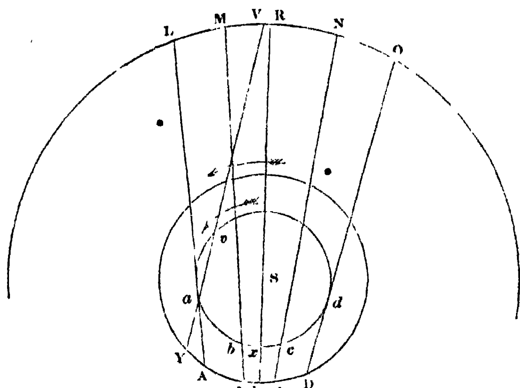
211. Of the planets in the above list, those which move in orbits between the sun and the earth are called *inferior planets*, while those beyond the earth are called *superior planets*. And we propose, before giving an account of each separate planet, to explain their *apparent* motions in the heavens, as seen from the earth, and afterwards to show by what means their *real* motions round the sun can be deduced from observation, and the elements of their orbits calculated.

212. We commence with the inferior planets, and we will take Venus as our example, this planet being familiar to all our readers as the most beautiful object in the heavens next to the moon. When the light is fading on a fine evening we see her shining in the west with a light incomparably superior to that of any of the other planets, and for some time she seems to retain a fixed position in the heavens. Her brilliancy increases as she approaches the sun, that is as she appears on each successive evening at sunrise nearer to the horizon, till she becomes lost in the sun's rays, and is missed for a time. After this if any of our readers would take the trouble to rise before the sun, they would see her shining with the same brilliancy in the east, and on successive mornings, would find her at a still greater distance, and rapidly separating from the sun. After a considerable time she would be found to become stationary again, and then, decreasing in brilliancy, to move in the contrary direction to meet the sun again, and, after a time, she would be for the second time lost in his rays. We will now illustrate, by means of a diagram, these oscillating motions, and show how they are natural geometrical consequences of her motion in an orbit smaller than that of the earth.

In the annexed figure let the inner circle represent the orbit of Venus; the middle one that of the earth; and the outer one the circle of the heavens. The directions of the arrows represent the directions of orbital motion round the sun, and of *direct* motion (that is, motion in the order of the signs of the zodiac) in the heavens.

Imagine Venus to come from *v*, moving in the direction *v a b c d*, while the earth is moving from ♈ in the direction ♈ A B C D. When Venus is at *v* she will be seen projected on the heavens at V, and when she is at *a*, (the earth being at A,) she will appear at L, having appeared to describe the *direct* arc V L. At *a*, however, she is moving in the direction of the line joining her and the earth,—this position

is called her point of greatest elongation from the earth and sometime after arriving at this point she will become stationary. The stationary point is determined by the consideration that the resolved parts of the motions of the planet and the earth perpendicular to the line joining them, must be equal. But after this, while she is still approaching the earth, and therefore becoming more

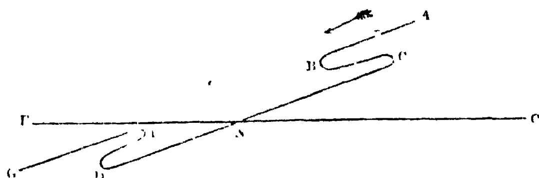


brilliant, if we take *a b*, A B, for arcs described by her and the earth in equal times, *a b* being considerably larger than A B, she will be seen at M, having appeared to retrograde through the arc L M. All this time she is to the east of the sun, or is visible in the evening, but after a time her motion will gain upon the earth's motion, till, arriving at *x*, she will pass the meridian at the same time as the sun, or be in inferior conjunction, and nearest the earth: she then passes to the other side of the sun and becomes a *morning planet*, and separates from him rapidly till she comes to *d*, before which she is again stationary; after this she proceeds towards *superior conjunction* with the sun, when she is at her farthest distance from the earth. The cycle of changes is then renewed in the same order.

213. It is evident from what has been said above, that Venus, as seen from the earth, never separates very far from the sun, but describes small arcs in the heavens, sometimes going a certain distance to the east and sometimes to the west of him. If the earth were quite stationary, this angle would be determined by drawing tangents from it to

her orbit, but the earth's motion modifies the apparent separation, and she sometimes passes the meridian as early as eight o'clock in the morning, and sometimes later than three in the afternoon, the equation of time conspiring to make the angular separation measured in time appear greater than it really is.

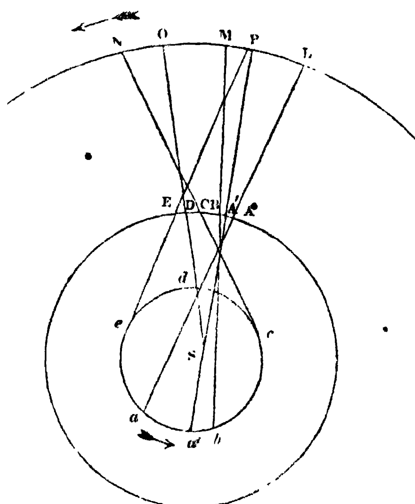
214. The apparent motions of the superior planets are different from those of the inferior, and admit of an equally simple explanation. If we were to map down the projected motions of any one of them, Jupiter for example, from year to year, we should find the projected path



to be something like that represented in the figure, where a straight line represents the projection of the ecliptic. The motion would appear to be *direct* through A B, stationary at B, retrograde through B C, then stationary at C, and then again direct through C D, crossing the ecliptic at N, and so on. It would appear in fact to describe a zigzag path very little inclined to the ecliptic, sometimes retreating and sometimes advancing, but on the whole gaining in direct motion. At certain times, for instance, we should find it in opposition with the sun, that is, the sun and planet would be on the opposite side to the earth; and at such times the *geocentric* and *heliocentric* longitudes (that is, the positions with regard to the first point of Aries measured from the centre of the earth and the centre of the sun) are the same. The observed arcs therefore obtained by comparing the observations made at successive oppositions will give the real advance of the planet in its orbit, notwithstanding its intermediate retrogradations.

215. We will now see how these motions are explained on the supposition of Jupiter describing an orbit exterior to that of the earth. Illustrating by a diagram—as before—let the inner circle represent the orbit of the earth, the middle one that of Jupiter, (the disproportion of the orbits is, for convenience, not represented,) and the outer one a section of the sphere of the heavens.

Let the earth and planet be at the same time at *a* and A, and, while the earth describes the arc *a b*, let the planet describe A B; the places of the planet will then be referred to the points L and M. It will therefore have appeared to describe the arc L M in the order of the signs, or with direct motion, and in this time it will have passed



conjunction with the sun at some intermediate position, A'. Hence on either side of conjunction the apparent motion of a superior planet is direct. Let now *c* and C be positions of the earth and planet when C *c* is a tangent to the earth's orbit, the planet will evidently then appear stationary soon after passing N; but, after arcs *c d*, C D, have been described by the earth and planet, the latter will be referred to the point O, having appeared to move backwards or to retrograde through N O. It will then become in opposition with the sun (that is, the planet and sun will be on opposite sides of the earth at an interval of 180° of geocentric longitude), and will continue to retrograde till it arrives at E, the earth being at *e*, where the line joining it and the earth is again a tangent to the earth's orbit, before passing which point it is again stationary. It will then begin to move *directly*, or according to the order of the signs, and so on for another revolution of the earth.

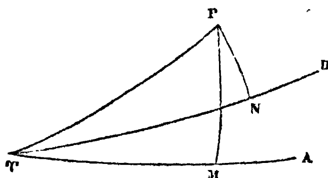
216. From the above considerations we derive the following rule :—

The inferior planets retrograde before and after inferior conjunction, and move directly in the rest of their orbits: the superior planets move directly before and after conjunction, and retrograde before and after opposition.

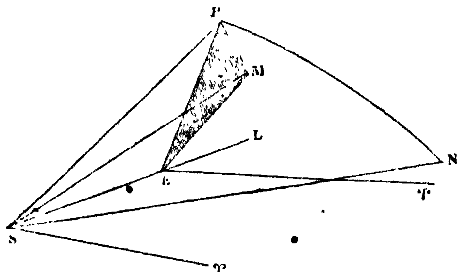
217. We will now show how the places of the planets as referred to the sun can be obtained from the geocentric observations.

We must again remind the reader, that the observations made with the fixed instruments of an observatory are those of Right Ascension and North Polar distance, and that these observations, before they can be used, require corection for refraction, parallax, aberration, and the motion of the equinox. They may then be considered as made at the earth's centre, and referred to a fixed equinox and a fixed equator. Now the position of the ecliptic with regard to the equator is known with the utmost accuracy, being derived from observations of the sun made near the equinoxes and solstices. At this present time, for example, the mean value of the obliquity is nearly 23° 27½′, and has a small secular diminution amounting to 0·457″ per year. It is then easy to reduce to the ecliptic the observed positions of the body referred to the equator, that is, to convert the observed Right Ascensions and North Polar distances to geocentric longitudes and ecliptic polar distances, or to geocentric longitudes and latitudes.

Thus, if ♈ A and ♈ B be portions of the equator and ecliptic, and P the position of a planet, then, if perpendicular arcs P M, P N be drawn to the two planes, ♈ M and P M will be the Right Ascension and declination, and ♈ N and P N the geocentric longitude and latitude of the planet, and it is a very easy problem in spherical trigonometry to deduce the latter in terms of the former, and of the known obliquity N ♈ M. We have then the place of the body referred to the plane in which the relative motions of the earth and sun are performed ; and the next process is to deduce the heliocentric longitude and latitude from the geocentric, that is, to refer the body's place to the sun, which is its proper centre, instead of the earth.



218. Let S, E, and P, be the positions of the sun, the earth, and a planet at a given time when the geocentric place of the latter is known; draw S ♈, E ♈ towards the vernal equinox, parallel to each other in the plane of the ecliptic, and P M perpendicular to the plane, and join S M, E M; then the



angles M E ♈ and M S ♈ will represent the geocentric and heliocentric longitudes, and P E M and P S M the corresponding latitudes. Then the geocentric quantities, that is, the latitudes and longitudes as seen from the earth's centre, being found from observation, the heliocentric longitude and latitude, and the distance of the planet from the earth, can easily be expressed in terms of them, and of the elements of the planet's orbit.

219. We have before had occasion, in treating of the solar motion, to mention these elements, but we will proceed now to speak of them more particularly.

It is plain that the position of the planet (to whatever points or planes we may refer it) will depend

First, upon the position of the plane in which it performs its orbit.

Secondly, Upon the magnitude, figure, and position of the ellipse which it describes in this plane.

Thirdly, Upon its position in the circumference of this ellipse.

Now, referring, as usual, every thing to the plane of the ecliptic,—the position of the plane of the orbit will be defined by the *longitudes* of the points or *nodes* where it meets the ecliptic, and by its angle of inclination to the ecliptic: the position of the major axis or lines of *apsides* of the ellipse will be defined by the longitude of the perihelion or aphelion: the magnitude and form of the ellipse will be defined by the semi-major axis, or mean

distance as it is called, and by the excentricity, or ratio which half the difference of the greatest and least distances bears to the mean distance; and, finally, the place of the planet in the ellipse will be known by means of its known position or longitude at some given time or epoch, generally called, for brevity, the epoch of longitude.

Hence the *six* quantities or elements which determine a planet's position are:—the *longitude of the node*, and the *inclination of the orbit*, the *longitude of the perihelion or aphelion*, the *mean distance*, and the *excentricity of the orbit*, and finally the *epoch of mean longitude*. To these may be added the *argument of latitude* or the angular distance of the planet from its node, measured along the orbit, which can be computed immediately from the node and inclination.

220. The way in which these elements must be treated differs considerably for the old or well known, and for newly discovered planets. For the former, such as Mercury, Venus, Mars, Jupiter, and Saturn, which were known and observed in ancient times, their elements were found and repeatedly corrected by observations made at long intervals of time, and by taking advantage of such circumstances of their motion as were most favourable for the discovery of particular elements. For example, the mean motion or sidereal period can be discovered with very great accuracy by observing the successive passages of the body through its ascending or descending node. For by reducing the observed right ascensions and declinations to geocentric latitudes and longitudes, it can be found when the latitude is about to become *nothing* or the body is about to cross the ecliptic, and, by observations made before and after this period, the exact moment at which it was in the node can be found by simple proportion, its motion in latitude being at this time tolerably uniform for short intervals. Hence the longitudes of the ascending and descending node will be known also. The inclination of the orbit to the ecliptic can also be found by observing when the latitude arrives at its maximum. We have also shown how the dimensions of the solar orbit have been accurately measured by means of the transit of Venus over the sun's disk, and this, in connexion with Kepler's Third Law, gives at once the dimensions of all the orbits, when the periods of revolution are known.

Now, by the geometrical relations existing between the

geocentric and heliocentric latitudes, longitudes, &c., in the preceding figure, we can with great ease express the argument of latitude in terms of the node and inclination, and of the geocentric longitudes of the planet and the sun; secondly, the distance of the planet from the earth can be expressed in terms of the sun's distance from the earth, the argument of latitude, and some of the above elements; and lastly, the radius vector of the planet, or distance from the sun, can be found in terms of previously known quantities, as also the heliocentric longitude and latitude. By such means a series of observed distances from the sun in the elliptic orbit will be obtained, and by assuming an ellipse of a certain excentricity, and with a certain longitude of the perihelion which shall nearly represent all the distances, an approximation will be made to the remaining elements. These elements will then be corrected by computing the heliocentric places of the planet corresponding to the times of other observations, and comparing them with the heliocentric places deduced from geocentric observations in connexion with the assumed elements. Equations of condition will then be obtained for determining the errors of the assumed elements to any degree of accuracy.

221. For unknown planets, every geocentric observation will by a somewhat similar treatment give two equations subsisting between the known geocentric quantities and the symbols expressing the elements. Hence three observations will give six equations, sufficient for determining the *six* elements of the orbit. Thus, in the greater number of instances of the recent discoveries of the small planets between Mars and Jupiter, no sooner were three complete observations announced than one or more of the zealous and able astronomers attached to the various observatories at home or on the continent have immediately furnished elements of the orbit, and *ephemerides*, that is, tables of daily right ascensions and declinations computed from the elements thus found, for the use of astronomers who might wish to follow the planet and make more numerous observations of its position.

We cannot expend more space upon this rather difficult subject, and we must devote the remainder of this chapter to the principal peculiarities of the separate planets of the solar system.

222. The planet MERCURY is too near to the sun to

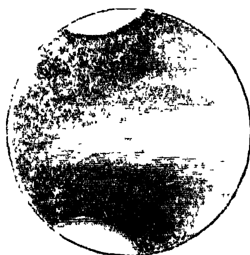
allow us to make any very accurate observations of the shape and other peculiarities of its surface. It has certainly no considerable ellipticity, that is, it is very nearly round, as the author has ascertained by micrometrical measurement. Its apparent diameter varies from 5″ to 12,″ and its real diameter is about 3200 miles. From doubtful observations of some spots on its surface it is supposed to revolve on its axis in about the same period as the earth, and in the same direction. Its mass is rather less than one-twelfth part of the earth, and its mean density is rather greater than that of the earth. The force of gravity at its surface is about one-eighth of that at the surface of the earth. It exhibits phases like the moon, and from a similar cause. The excentricity of its orbit is large, being about 0·205, and the inclination is also large, being 7°. Its excursions on each side of the sun do not much exceed 18°. Supposing all our heat to come from the sun, it is calculated that the mean heat in Mercury is above that of boiling quicksilver, and even near his poles water would always boil. The sun's diameter would appear from his surface more than twice as large as from the earth.

223. VENUS is a far more interesting planet, and, on account of her greater distance from the sun, admits of more frequent and accurate observation. Her light in a telescope is, however, so dazzling and brilliant, that, by exaggerating all the defects of the telescope, physical observations of the surface are difficult. By means of the rotation of spots imperfectly seen on her surface by the German astronomer Schroeter, the time of rotation on her axis is imagined to be rather less than that of the earth, and also from west to east. The phases, which are exactly similar to the moon's, only with much longer periods, are very interesting when viewed through a telescope, and her brightness is such as to be seen occasionally at midday, when her position with respect to proximity to the earth is most favourable. The excentricity of the orbit is very small, and the inclination does not amount to 3½°; her apparent diameter sometimes amounts to 1′, and her real diameter is about 7800 miles. She seems to be surrounded by an atmosphere which may probably mitigáte to her inhabitants, if she have any, the intense heat of the sun, which yet must be far too great for the existence of plants or animals like those existing on the earth. Her mass is rather greater, and her density rather less than that of

the earth. Hence the force of gravity at her surface is about the same as on the surface of the earth. The reader will not forget the important service rendered to astronomers by this planet, in the ascertaining of the solar parallax, by her transits across the sun's disk.

224. Venus being so near the earth, it might be expected that she would produce disturbances both in the orbit of our own planet and of its satellite the moon. Such is in fact the case. An inequality, having a very long period, in the motions of the earth and Venus was discovered many years ago by Mr. Airy, and a curious inequality in the lunar orbit, arising from the attraction of Venus, was not long ago discovered by M. Hansen, astronomer at Gotha.

225. MARS, the nearest of the planets exterior to the earth, that is, of the superior planets, offers more points of similarity than any of the others. The excentricity of the orbit is, however, considerably greater, being rather less than $\frac{1}{10}$. The inclination to the ecliptic is 1° 51′. His diameter is about 4100 miles; when in opposition to the sun, that is, nearest to the earth, his apparent diameter is about 20″; and the ellipticity as resulting from six sets of measures carefully made by the author at the recent opposition in the beginning of the present year, 1852, is rather greater than $\frac{1}{60}$; while six sets of measures made at Greenwich, at a much more favourable opposition of the planet in 1845, give for the ellipticity $\frac{1}{52}$. It is evident, therefore, that a tolerably close approximation has been made to the value of this minute quantity. Assuming the mean value



of the diameter to be 20″, and the ellipticity $\frac{1}{55}$, the quantity thus found to be accurately measurable with the double image micrometer is 0″·36. The disk is so well seen through good telescopes, that rude maps of his surface have been

drawn, in which something like a vague delineation of seas and continents is exhibited. The colour of the darkest part is that of a brownish red, and near the poles is a zone of white, indicating the existence of snow in large quantities. In the accompanying engraving, the distinctness of the white spot near the south pole is exaggerated, but a remarkable one existed when the drawing was made. His climate must be considerably colder than our own, but, as the inclination of his axis to the ecliptic is nearly the same as that of the earth, and the period of diurnal rotation ($24^h\ 37^m$) only a trifle larger, the changes of the seasons must be very similar to our own, only with much greater variations. His mass is about one seventh part of that of the earth, and his density only a trifle smaller. The force of gravity at his surface is about $\frac{1}{2}$ of that at the surface of the earth. It seems tolerably certain that Mars is surrounded by an atmosphere of considerable density.

226. Of the small planets between Mars and Jupiter we know very little, except their geometrical positions. Some of them are exceedingly minute, and their surfaces are not much larger than a large estate. Both Ceres and Pallas exhibit traces of nebulosity round them indicating atmospheres, and the same circumstance was noticed at the discovery of some of the recent ones. The brightest of them is Vesta, which appears when nearest to us like a star of the 5-6 magnitude, and Pallas is also remarkable for the vivacity of her light; but the others vary generally from the 7th to the 13th magnitudes, according to distance. The discovery of the recent ones is a very honourable characteristic of our age, and reflects great credit on their discoverers, particularly on Mr. Hind, and M. De Gasparis of the Observatory of Naples. Each of these gentlemen has discovered four, and will probably not be content till several others have been discovered. The original discovery of Ceres by Piazzi, had its origin in a curious speculation, arising out of the failure of Bode's law at one point between Mars and Jupiter, which led to the idea of a large planet having been shattered to pieces. An Association of Astronomers determined therefore to search in the most likely parts of the heavens for the fragments, and were soon rewarded by the discovery of the four with which our readers are familiar, viz. Vesta, Juno, Ceres, and Pallas, near the beginning of the present century. It was not till the year 1845, that the next in order of discovery, viz. Astræa, was found by

M. Hencke, of Driessen.* Since that time the discovery of the remaining ones has gone on with tolerable continuity, and we may still hope for several others to be added to the solar system.

When the orbits of all of them are known with greater accuracy, a projection of them will show better whether the original idea of the explosion of a large planet is tenable or not. Astronomers at present are rather divided in opinion on this subject.

227. We should not omit to mention, in connexion with this subject, the zodiacal star-maps constructed by Professor Bremiker, of Berlin, in which the positions of all stars down to the 10th magnitude within the zodiacal limits are represented, as these maps have not only materially assisted in the discovery of these small planets, but the planet Neptune was by means of them recognised immediately on the telescope of the large equatorial at Berlin being directed by Dr. Galle towards the position indicated to him by M. Le Verrier. A series of such charts of stars observed by Mr. Bishop and Mr. Hind, in the Regent's Park, is also in course of publication.

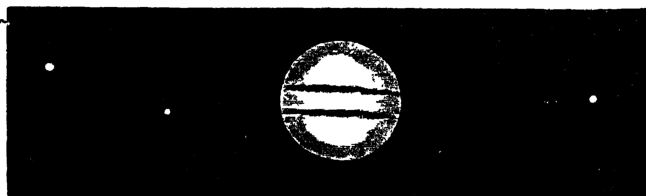
228. The orbits of these small planets differ from those of the large planets previously known chiefly in their inclinations, which are not included within the zodiacal limits, but are many of them very large, and this feature renders the computation of their perturbations by the large planets very intricate and difficult. A method of calculation devised by the celebrated astronomer Hansen has, in a great measure, conquered this difficulty, and considerably extended the power of analysis in the computation of the places of the planets generally.

229. Jupiter, the largest, and in many respects the most important, of all the planets, is next in order of distance from the sun; the sidereal period is 4332·584 days, and his synodic period, that is, the interval between his successive conjunctions with the sun, is 398·867 days. The inclination of his orbit to the ecliptic is 1° 19′ nearly, and the excentricity is 0·04818. His mean distance from the sun is rather more than five times that of the earth.

230. The shape of this stupendous globe is plainly elliptical, even to a casual observer, when viewing him

* See *Notices of the Royal Astronomical Society,* vol. vii. p. 27, for remarks on the discovery of this planet.

through a good telescope, and the ellipticity as deduced from a great number of careful observations at Greenwich, made with a double-image-micrometer, is about, $\frac{1}{16}$. The time of revolution on his axis, as determined by certain spots at times visible in his atmosphere, is $9^{h}\ 55^{m}\ 50^{s.}$, and the ellipticity calculated from this time of rotation, on the suppo-



sition of the original fluidity of the globe, corresponds pretty accurately with the observed ellipticity. The angular mean diameter, or axis major of this ellipse, is about 40″, which corresponds to a real diameter of about 87,000 miles, nearly eleven times the diameter of the earth. The mass is about $\frac{1}{1049}$ (that of the sun being unity) as determined by Mr. Airy from the observed elongations of the fourth satellite (see page 82); that is, the mass is upwards of 370 times that of the earth, but the density is not quite one quarter of that of our planet. The force of gravity at the surface is about $3\frac{1}{4}$ times that of the earth.

231. The disk of Jupiter is crossed by dark bands, or *belts* as they are generally called, above and below the equator, as is denoted in the figure. These belts suggest the idea of, and there is little doubt that they owe their origin to, disturbances in the atmosphere surrounding the planet. They vary much at different times in breadth and situation, and are evidently not of a rigid or permanent character.

232. Jupiter is attended by four satellites or moons revolving round him from west to east, in the same way as the moon does round the earth, according to the law of gravitation. These satellites suffer eclipses on entering into the shadow of Jupiter, and are occulted or hidden when they pass behind his body; they are also observed to pass over or to *transit* his disk, and at such times their shadows can be seen like black spots passing along the disk. On entering on the body of the planet, they can be distinctly seen when near the border, by their superior

brightness, but they are lost sight of when approaching the centre: this proves most distinctly that the border is *shaded*, and indicates an atmosphere of some density. The times of the eclipses, when observed at different places on the earth's surface, determine directly the difference of longitude of the places, since they must occur at the same moment of *absolute* time, but the observations are given in terms of *local* time. It was also by means of eclipses of the satellites at times when Jupiter was at very different distances from the earth, that Röemer discovered the successive propagation of light, and determined its velocity.

233. To an observer on Jupiter, the first satellite would appear rather larger than our moon, the second rather more than half as large, the third rather larger than the second, and the fourth not half so large. Their actual diameters are about 2508, 2068, 3377, and 2890 miles.

234. The mean motions and times of revolution of the first three satellites are connected by a singular law; viz. that the angular velocity of the first added to twice that of the third is equal to three times the angular velocity of the second. And hence we might easily prove that if "from the mean longitude of the first added to twice that of the third there be subtracted three times the mean longitude of the second, the remainder will be a constant angle." This constant angle is found to be equal to 180°.

235. The above remarkable relation between the mean motions of the satellites leads to this interesting result, viz. that they cannot all be eclipsed at the same time; that is, that even in extreme cases Jupiter will never be deprived of the light of all his moons at once. We need do no more than indicate to the reader, without further remark, this proof of beneficent wisdom.

236. The first three satellites move very nearly in the plane of Jupiter's equator, in orbits very nearly circular. The inclination of the fourth to the equator is about 8°, and its excentricity is large. In consequence, when the fourth satellite is seen to pass across Jupiter's body or behind it, the apparent path is frequently very far from the centre. The writer of this little book remembers to have watched one such transit, when the satellite merely grazed the upper part of the disk, and was partially visible or projected beyond the body during the whole time.*

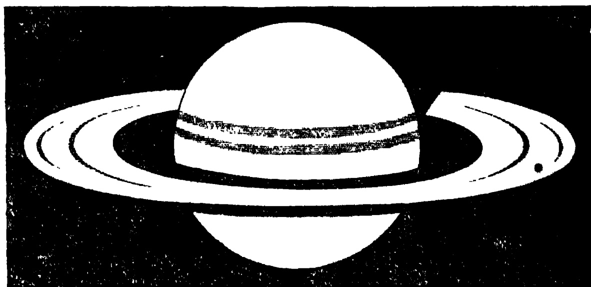
* See *Greenwich Observations for* 1844, p. 138.

237. The mass of Jupiter being so great, it might be naturally expected that his influence would be considerable in disturbing the other planets, and this is really the case. But the most curious circumstance in the disturbances produced by him is the reciprocal effect produced in the motions of himself and Saturn, his neighbour in the heavens, and almost rivalling him in bulk. An equation or disturbance, having a very long period of about 918 years, is produced by the mutual actions of these immense bodies, of such a nature that if one be, by the disturbance, put *before* its mean place, the other will be *behind* its mean place. At present, the motion of Saturn is accelerated by the disturbance, and that of Jupiter retarded, but in the seventeenth century, the circumstances were reversed, and Saturn was retarded while Jupiter was accelerated. This inequality, known by the name of "the great inequality of Jupiter and Saturn," is of such a magnitude, as at its maximum to advance or retard Saturn by about 0° 49′ in longitude, and to retard or advance Jupiter by about 0° 21′. We have not space, nor does it fall within our plan, to trace the physical cause of this remarkable effect of disturbing action, but the reader may be assured that a general explanation can be given in much the same way that has been used for the inequalities of the moon. The more advanced reader may consult Sir J. Herschel's "Outlines of Astronomy."

238. The next planet is SATURN, not inferior in general interest to Jupiter, and of equal importance in the planetary theory. His most remarkable appendage is a luminous ring, by which he is generally seen to be surrounded. This ring is a solid opaque substance, as is proved by the shadow of it which is thrown on the body of the planet. By its parallelism with the belts with which Saturn as well as Jupiter is striped, it appears that the axis of rotation of the planet is perpendicular to its plane, and observations of spots on the surface of the planet give for the time of his rotation about $10^h\ 29^m\ 17^s$. The rotation of the ring itself has been detected by means of the observations of portions of its surface less bright than others, and its period of revolution is $10^h\ 32^m\ 15^s$, which is very nearly the time in which a satellite would revolve round a body of the known mass of Saturn, at the distance of the middle of the ring from the centre. Indeed, without a rapid revolution it can be proved that the ring could not be

maintained in equilibrium, but must inevitably fall on the body. The ring, till lately, was known to consist of two distinct portions, separated by a dark interval easily seen in good telescopes, but recently a discovery has been made of the existence of a *third* ring inside the other two. This ring, which requires the best state of the atmosphere, and a very good telescope, as well as an experienced observer, to be rendered visible, seems formed of a substance totally different from that composing the other two, and reflects light so imperfectly that its existence was unknown till near the end of the year 1850, when it was almost simultaneously discovered, in America by Mr. G. P. Bond, of the Cambridge Observatory, Massachusetts, and in England by Mr. Lassell, when on a visit to the Rev. W. R. Dawes, at Wateringbury, near Maidstone, where the latter gentleman has established an observatory.

239. The annexed figure is copied from a drawing made by Mr. Dawes of the appearance of Saturn after the existence of the inner ring had been well ascertained, and besides exhibiting the semi-reflective ring which it was principally intended to show, exhibits also a delicate subdivision of the outer ring, visible only near the extremities, but of whose reality Mr. Dawes quite convinced himself by repeated glimpses of it caught at favourable opportunities. It had been previously suspected by astronomers



that the outer ring had at least one sub-division, but the matter seems now by this evidence to be put beyond the reach of doubt.

Mr. Lassell's account of his discovery of the inner ring is as follows:—*

* See *Notices of the Royal Astronomical Society*, vol. xi. p. 21.

"After surveying the planet for some time, I was struck with a remarkable phenomenon, which I shall proceed, as well as I can, to describe. It appeared as if something like a *crape veil* covered a part of the sky within the inner ring. This extended itself half way between what I should have formerly considered the inner edge of the inner ring and the body of the planet, while there was a darker, ill-defined boundary line separating this crape-like appearance from the solid body of the inner ring. There was an exceedingly thin line, or shadow, running along the southern edge of the northern portion of the ring where it crossed the planet, and the line seemed somewhat broader at each end, where it touched the limbs of the planet. Mr. Dawes had previously drawn my attention to the appearance of this line before scrutinising the planet."

240. Saturn is now known to be attended by eight satellites, to which have been given the mythological names Mimas, Enceladus, Tethys, Dione, Rhea, Titan, Hyperion, Iapetus. These satellites were all of them seen at the same time by Mr. Lassell, on the evening of November 21, 1850,* with his reflecting telescope, but opportunities seldom occur, and several of them require not only telescopes of great power, but a very favourable state of the atmosphere to render them visible. Of the above, the outermost, Iapetus, was discovered by Dominic Cassini, in 1671. Its distance from the planet is nearly three times as great as that of any of the others, and its orbit is considerably inclined to the plane of the ring (by 12°). It is also remarkable for the diminution of its light in certain parts of its orbit, and this obscuration occurring constantly on the same side of Saturn as seen from the earth, it is proved with tolerable certainty that it revolves on its axis in the precise time of its revolution round Saturn. Hyperion, the next satellite, reckoning inwards towards the planet, was discovered recently in the month of September, 1848, simultaneously by Professor Bond at Cambridge Observatory, Massachusets, and by Mr. Lassell at his observatory at Starfield, near Liverpool, and affords another good instance of the rivalry existing between the eminent observers of the present time.

241. Titan is the largest and best known of the satellites, and was naturally the first discovered. It was first seen by

* See *Notices of the Royal Astronomical Society*, vol. xi. p. 19.

the celebrated Huyghens in 1655, March 25, and is supposed to be not much inferior in size to the planet Mars.

242. The remaining satellites are much more difficult to observe, especially the two innermost, which just skirt the ring, and at the epoch of their discovery in 1789, by Sir William Herschel, they were seen to thread like beads the almost indefinitely thin fibre of light to which the ring was then reduced.*

243. It is proper to mention that the well-devised mythological designations of the satellites are due to Sir J. Herschel, and were proposed for the purpose of avoiding the confusion arising from the order of discovery not coinciding with the order of distance, so that the numerical nomenclature, like that employed for the satellites of Jupiter, was quite insufficient to distinguish them.†

244. Having thus discussed, as fully as our space permits, the particulars relating to the ring and satellites of Saturn, we will now give in few words the most important facts relating to the planet itself. The shape is elliptical, with an ellipticity of about $\frac{1}{9}$, as is proved by measures made by the author of this little work at the Royal Observatory, during the time of disappearance of the ring in 1848.‡ It was thought by Sir William Herschel, and the opinion, till recently, has been generally entertained by astronomers since his time, that the shape of the planet was oblong, but not elliptical, something like a parallelogram with the corners rounded off. It has, however, been proved by the above measures, confirming those made several years since by Bessel, that such is not the case, but that the shape is that of an exact spheroid of revolution of considerable ellipticity.

245. The inclination of the orbit of Saturn is 2°30′ nearly, and the excentricity 0·0561; the mass is 111 times that of the earth, the diameter is 79,160 miles, and the force of gravity at the surface is about $1\frac{1}{8}$ time that at the surface of the earth. The sidereal period is 10759·22 days. The diameter of the outer edge of the interior ring is about $2\frac{1}{4}$ times that of the equatorial diameter of the body.

* See *Outlines of Astronomy*, p. 336.

† See *Sir J. Herschel's Results of Astronomical Observations made at the Cape of Good Hope*, p. 415, published in 1847; where the nomenclature now in use was first proposed. In the foot-note, the following remarkable passage occurs: "Should an eighth satellite exist, the confusion of the old nomenclature will become quite intolerable."

‡ See *Memoirs of the Royal Astronomical Society*, vol. xviii.

We will conclude this account of Saturn by an eloquent passage from Sir J. Herschel's book, which we have so frequently had occasion to refer to.

"The rings of Saturn must present a magnificent spectacle from the regions of the planet which lie above their enlightened sides, as vast arches spanning the sky from horizon to horizon, and holding an almost invariable position among the stars. On the other hand, in the regions beneath the dark side, a solar eclipse of fifteen years in duration, under their shadow, must afford (to our ideas) an inhospitable asylum to animated beings, ill compensated by the faint light of the satellites. But we shall do wrong to judge of the fitness or unfitness of their condition from what we see around us, when, perhaps, the very combinations which convey to our minds only images of horror, may be, in reality, theatres of the most striking displays of beneficent contrivance."

246. The planet URANUS was discovered by Sir William Herschel, on March 13, 1781, in the course of a general review of the heavens, being detected by its disk under a high magnifying power. At the author's request, Professor Challis obligingly measured the planet some years ago, with a double-image-micrometer attached to the telescope of the great Northumberland telescope,* for the purpose chiefly of discovering whether it had any sensible ellipticity, which the author suspected from some measures of his own made with a far inferior telescope. The result was that the ellipticity is too small to be measurable, and the apparent diameter about 4″. Its real diameter is about 35,000 miles.

247. The inclination of its orbit to the ecliptic is very small, being only 46½′, and the excentricity 0·0467. The sidereal period is 30686·82 days.

248. The satellites of Uranus, as might be expected, are exceedingly faint objects, and difficult to be observed even with very powerful telescopes; and there exist still considerable doubts respecting their positions, and even respecting their number. Sir William Herschel observed two satellites, which are considerably brighter than the rest, about six years after the discovery of the planet, and obtained with tolerable accuracy their distances and periodic times. Several years afterwards he announced the discovery of four other satellites in the *Philosophical Transactions* for

* For a description of this celebrated telescope, see our article on Observatories in "London Exhibited."

1798, and, in a memoir printed in the *Philosophical Transactions* for 1815, he gave the results of his observations to that date. Of the six satellites thus presumed to exist, the comparatively brighter ones he reckoned to be the second and fourth, counting outwards from the planet, and their periods he calculated to be about $8^d\ 16^h\ 56^m$ and $13^d\ 11^h\ 9^m$. The periods of the others, according to his estimations, were approximately $5^d\ 21^h$, $10^d\ 23^h$, $38^d\ 2^h$, and $107^d\ 17^h$. Now, since the time of Sir William Herschel, the positions of *none* of the four satellites last mentioned have been verified, and Sir John Herschel, who bestowed considerable attention on the system of Uranus, was enabled only to observe the two brighter ones.

The first addition to our knowledge was made at the close of the year 1847, by Mr. Lassell and Mr. Otto Struve, the former by means of his reflecting telescope, and the latter by means of the large refractor of the Pulkova Observatory. In the *Notice of the Royal Astronomical Society* for 1848, January 14, is a communication from each of these astronomers, announcing the discovery of at least one satellite additional to the second and fourth of Sir W. Herschel. Mr. Lassell observed a satellite evidently interior to Herschel II. on several nights, and on one occasion he observed an additional one. The positions of the objects observed by him were also always on the north side of Uranus. M. Otto Struve observed, on the contrary, an object always on the south side of the planet, and there is only one day of observation common to himself and Mr. Lassell. In a paper inserted in the *Notice of the Royal Astronomical Society* for 1848, March 10, Mr. Dawes discusses very elaborately all the observations of both astronomers, and comes to the conclusion that M. Struve's satellite is not identical with either of Mr. Lassell's, but that it is probably identical with an object observed by Mr. Lassell in 1845, on September 27, and that its period, deduced by comparison of these distant observations, is about $3^d\ 22^h$. He also finds the period of the object observed more than once by Mr. Lassell to be about $2^d\ 2\frac{2}{3}^h$, by comparing it with an object observed by Mr. Lassell, 1845, October 5.

This was the state of our knowledge of the satellites till near the end of the preceding year, 1851, when Mr. Lassell announced the discovery of two satellites, whose periods, by observations which admit of no doubt, are about $2^d\ 12^h$ and $4^d\ 3^h$.

The result, then, of the whole discussion seems to be, that at present at least *four* satellites are known, and that their periods and distances are ascertained with tolerable accuracy, and that though several others undoubtedly exist, yet we have, at present, no observations before us to give any definite information about them in confirmation of Sir W. Herschel's results.

The satellites move in orbits very different from those of all the other planetary systems, the planes being nearly perpendicular to the ecliptic, and their motions in their orbits being retrograde.

249. The mass of Uranus has always been a subject of great interest with astronomers, and is now of much greater on account of its action on the newly discovered planet Neptune. This element is of very difficult determination by means of the revolutions of the satellites, owing to their faintness. The most recent determination is that made in 1848 by M. Otto Struve, which gives for the mass $\frac{1}{17918}$ part of that of the sun, or rather more than twenty times that of the earth. Hence the force of gravity at his surface is rather greater than that at the surface of the earth.

250. The last, and from the circumstances attending its discovery, the most interesting planet which we have to mention, is NEPTUNE. The discovery is equally due to two mathematicians, viz. M. Le Verrier of Paris, and Mr. Adams of our own University of Cambridge.

It had long been known to astronomers that the observed place of the planet Uranus disagreed with its place calculated from the well-known elements of its orbit by a very large quantity, and that this disagreement was increasing from year to year. At the present time, for example, the R. A. calculated in the ordinary way from the elements differs from the observed R. A. by more than 11^s, and the Polar Distance by more than $1'$, an amount of difference far exceeding any error in the calculation of its disturbances arising from the known planets, or from any failure in the theory of its motion generally. The idea of disturbances arising from some *unknown exterior planet* had been suggested long before the error attained this amount, and was the most probable solution, but some distinguished astronomers, in their uncertainty about the cause, began seriously to question the *exact* accuracy of the law of gravitation, that is, of bodies mutually attracting each other according to the inverse square of the distance. At all events, no one had

courage to attack the problem under this point of view, and to endeavour to find the intruder and the disturber of our system. It appears, however, from a memorandum in the journal of Mr. Adams, that he had for some years formed the resolution of endeavouring to solve the problem analytically as soon as he should get leisure from his academical engagements. M. Le Verrier had also been engaged in a laborious investigation of the elements of Uranus according to the existing theory, and had succeeded in finding one or two trifling additional terms of perturbations from known planets, and in discovering some inaccuracies in Bouvard's tables. He had, however, found nothing that would at all explain the enormous difference existing between theory and observation. The two geometers became thus, quite independently and without any knowledge of each other either personally or otherwise, engaged in the laborious problem till then quite despaired of, of finding *the elements of a disturbing planet by means of the disturbances produced upon the disturbed planet*. The direct problem is difficult enough to try ordinary industry and patience, but the intricacies of the inverse problem were enough to make the boldest analyst and computer shudder.

We cannot give any adequate idea of the labour and difficulty actually encountered, except by stating that all the elements of the sought planet (except the mean distance which it was necessary to assume arbitrarily) must come into the equations of condition, as well as all those of the disturbed planet, or rather their corrections, since the latter elements obtained from the ordinary theory must be necessarily false. Both mathematicians did, however, arrive at about the same time at a complete relation, and both, in the autumn of the year 1846, furnished elements of the planet sought for. Le Verrier, from his elements, calculated a place of the planet, and so confident was he in the accuracy and certainty of his result, that he wrote to his friend Dr. Galle, of the Royal Observatory of Berlin, requesting him, on the night of the receipt of the letter, to direct the telescope of the large equatorial of that establishment to the indicated spot in the heavens, and giving him full assurance of finding the planet. This letter reached Dr. Galle in 1846, Sept. 23rd. The Berlin Astronomer, on searching as directed, found a sta of about the 8th magnitude, which did not exist in Bremiker's star charts, which we have previously had occasion to mention, and

which, therefore, afforded strong presumption of being the body sought for, and the fact was confirmed by the first night's re-observation. In the meantime, the researches at Cambridge had proved in the English mathematician and astronomer an equal degree of merit, though of course the glory of the first discovery indisputably belongs to Le Verrier and Galle. Professor Challis, the Director of the celebrated Observatory at that University, had for some time been engaged in a laborious and well-arranged sweep of that part of the heavens which Mr. Adams's researches had pointed out as the most likely, and it is a curious fact that, before its actual discovery and recognition at Berlin, it had been twice observed by that unwearied astronomer. To explain the fact of its being observed, and yet not recognised, it is necessary to remind the reader that at this time, that portion of Bremiker's star charts, which included the portion of the heavens in question, had not yet reached England, so that Professor Challis had, in fact, to *construct one for himself* from the data afforded by his observations. If the result of each night's sweep had been pricked off on prepared maps immediately afterwards, the glory of the discovery would have rested with him and Adams, but the other laborious duties of the observatory prevented this being done, at the same time that the necessity of doing it was not so obvious before the discovery as it appeared afterwards.

The claims of the rival astronomers and mathematicians are evidently, on the whole, equal, and the planet would infallibly have been discovered during the autumn by each independently. At the same time the peculiar fame arising from the happy union of transcendent analytical skill and undoubting confidence in the result, must be conceded to Le Verrier; his whole process was happily conceived, carried out in the most masterly way, and immediately rewarded by his singular boldness in announcing his result and pledging his reputation on its certainty.

We could not pass by this wonderful discovery of the planet Neptune without thus much of notice. It exhibits, in the most striking way, the advance of astronomy in its theoretical development, and in its practical adaptation:—the mathematician, the observer, and the instrument-maker were almost equally needed in the solution of the great problem before us;—and if any one of these conditions had been wanting, Neptune might have remained still unknown.

251. Immediately after the discovery, the observations of the last century were ransacked to endeavour to find whether Neptune had been observed without knowledge of its planetary character, since this was of very great importance in the calculation of its real orbit from observation. The published observations of the French astronomer Lalande afforded no trace of such an observation, but, on consulting the manuscripts of these observations kept at the Observatory of Paris, two were found, made in the year 1795, on the 8th and 10th of May, which were suppressed in the printing, on account of the error presumed from their disagreement (the object being supposed to be a star), but which were really observations of the planet.

252. From these observations compared with those made immediately after the discovery, elements of the planet were computed by Professor Walker of the United States, which up to the present time represent its motions with perfect accuracy. These elements give for the sidereal period 164·6181 tropical years, for the excentricity 0·008 719, and for the inclination 1° 47′.

253. Soon after the discovery the planet was attentively watched by Mr. Lassell, M. Otto Struve, Mr. Bond, U. S., and by Professor Challis, for the discovery of any physical peculiarities that might be rendered visible in the powerful telescopes at their command. Mr. Lassell was rewarded by the discovery, in October, 1846, of a satellite, which M. Otto Struve, by subsequent observations, found to be inclined to the ecliptic at an angle of about 35°. All the observers, or the greater number of them, also agreed in describing the planet's appearance as elongated, and decidedly not circular, and affording very strong suspicions of its being surrounded by a ring like Saturn.

254. By observations of the satellites at Cambridge in America, and at Pulkova, two separate values of the mass of the planet have been deduced by Professor Peirce and M. Otto Struve. The values are respectively $\frac{1}{19840}$ and $\frac{1}{14496}$ of the sun's mass, and, as might be expected from results deduced from observations of so difficult an object as a scarcely distinguishable satellite, they differ materially.

255. Before concluding this chapter, it is necessary to devote a small space to comets, a class of bodies allied in some respects to the planets, but differing so much in many

important particulars as to make it necessary to treat of them separately.

256. COMETS, like planets, revolve round the sun evidently under the influence of the law of gravitation, for they all describe orbits identical with one of the conic sections, and they all obey Kepler's "Second Law" of the equal description of areas, as is evident from the accomplished predictions of the return of some of them, for which the calculations assumed the law.

The planets, however, always describe ellipses of very small excentricity; they all move according to the order of the signs or with direct motion, and the inclinations of the orbits of the large planets to the ecliptic are always small.

Comets, on the contrary, move in *parabolic* and even in *hyperbolic*, as well as in *elliptical* orbits; the inclinations of the orbits have all degrees of magnitude; and their motions in their orbit are as often retrograde as direct. Indeed, the greater number of those whose orbits have been calculated describe parabolas having the sun in the focus; that is, during the time of their visibility, the orbit, if it differs at all from a parabola, differs so slightly, that the difference cannot be detected by observation.* Very few of them have been ascertained to move in *hyperbolas*, but the fact is certain for some of them. A great many periodical comets, however, (that is, comets that circulate round the sun, and are observed, after their departure, to return to it again), moving in ellipses of various excentricities are known at present.

257. Now our readers will remember in our remarks on the planetary motions, that a body might describe either an ellipse, a parabola, or an hyperbola round the same centre of force, the particular orbit being determined by the velocity and the angle of direction made with the radius vector at any point. Thus, supposing a body moving in a parabola, suddenly to suffer some resistance which would materially lessen its velocity, it might describe an ellipse, and if its velocity were suddenly increased it would describe an hyperbola.

It is found also by telescopic observations, and by the very small effect produced on the motions of the planets when comets approach near them, as also by the very great disturbances which the motions of the latter suffer on such occasions, that the masses of all the known cometary bodies

* Our readers are assumed to know that an ellipse, when its major axis becomes infinite, degenerates into a parabola.

are exceedingly small, indeed of incomparably less density than our atmosphere, or even of any of the lighter gases that we are acquainted with. Thus the comet of 1770, called Lexell's comet, actually approached, in 1779, Jupiter, with a distance scarcely exceeding $\frac{1}{500}$ part of the distance of that body from the sun, and yet produced no perceptible disturbance on the satellites, though it is exceedingly probable that its own return to visibility by us was quite prevented by the great disturbances it experienced. Again, the texture of which even the head or densest part of a comet is composed is of so small a density that the faintest stars are seen through it, though a slight fog or vapour near the earth's surface will render invisible stars of the first magnitude.

Hence we can conceive, without any undue speculation on the nature or origin of these mysterious bodies, that, whatever were the original orbit in which a comet may have been moving, a near approach to any of the large planets might so effectually change its orbit, that before its discovery it might be moving in any one of the curves before mentioned.

258. We have mentioned above, that comets are of extreme tenuity, but we will add a few more remarks on their



physical appearance, which is extremely different in different specimens of these bodies, and even at successive returns of the same. Some have been seen of immense magnitude and brightness, visible even at noon-day, and attended by a tail

extending over 60° to 70° of the heavens. Generally in such cases the head consists of a brightish spot near the centre or *nucleus*, surrounded with a small circular nebulous mass or *coma*; beyond this is a small interval of clear space, and outermost of all is a luminous *envelope*, going round the coma like the head of a parabola, and generally extending in the direction of straight lines, so as to form two streams of light diverging for some distance, and sometimes reuniting. Frequently, however, the comet has no *tail* or train, and consists of a small nebulous mass with or without a nucleus, and frequently, in the cases of what are called telescopic comets, a small nebula of extreme faintness is all that is visible to experienced eyes. Again, of comets which have tails, the shape is exceedingly various. Some have one tail, others two or three at different angles, and some have been seen with still more. Generally the tail is curved towards the part of the heavens from which the comet has been moving, but its length and the general appearance of the comet is greatly affected during the time of approach to, and recess from, the perihelion passage, or passage through the point of its orbit nearest to the sun.

259. We will now particularise a few of the most remarkable comets that are known or suspected to be periodical.

260. Of comets of long period, the most remarkable is that known by the name of Halley's, from the circumstance of that illustrious geometer having predicted the return. On applying the principles of the theory taught by Newton with regard to these bodies, and calculating the orbits of several ancient comets, Halley found a remarkable coincidence in the elements of the orbits of comets which had appeared and been observed at nearly equal intervals of time in 1531, 1607, 1682, the latter appearance taking place in his own time. After mature consideration he alleged that these comets must be identical, and predicted the return about the year 1759. Clairaut, an eminent mathematician, undertook the calculation of the disturbances it would experience from the large planets, by which its return was delayed about six hundred days, and the comet returned again to perihelion according to prediction in the spring of 1759. Our readers may remember to have seen it again in the winter of 1835, when its return had been calculated with great certainty by several eminent mathematicians.

261. A great comet which appeared in 1680 is supposed to have a period of 575 years, and to be identical with

famous comets seen in the years 1105 and 575, and also forty-three years before the Christian Era, soon after the death of Cæsar.

262. Another great comet which appeared in 1556 is supposed so to have excited the superstitious fears of Charles V., Emperor of Germany, as to have influenced him in his abdication. The return of this comet in 1848 was predicted by Mr. Hind from elaborate calculations, but it falsified prediction; probably owing to some extraordinary perturbations which it may have experienced either in receding from, or approaching to, the solar system.

263. Another comet appeared in 1661 which is supposed to be identical with that seen as early as 243 A. D., and returning in 891, 1145, 1402, and 1532.

264. Another class of comets having much shorter periods, has been discovered during the present century. The first of these, discovered in 1819, is called Encke's comet, that astronomer (Professor Encke, of Berlin,) having devoted much time and attention to the study of its theory. It revolves round the sun in the short period of 3⅓ years, and since its discovery it has been observed several times on its return to perihelion. The author observed it at Greenwich in 1837, and at other times when it was an exceedingly faint and difficult object, but it differs greatly in its appearances at different times. By its means a most important and curious physical fact has been elicited. It is found that its periods of revolution are successively and equably shorter, or that the major axis of the elliptic orbit is slowly but steadily diminishing. No probable cause has been assigned for this anomalous fact, excepting the resistance which a very light gaseous body might experience in passing through a medium of sufficient tenuity to prevent any sensible resistance to the denser planetary bodies. Professor Encke has calculated its successive returns on this hypothesis, but the question does not seem to be quite set at rest.

265. Another comet of short period is the comet of Biela or Gambart, those astronomers having identified it at its apparition in 1826, with comets that appeared in 1772 and 1805. The time of revolution of this comet is about 6⅔ years. At its return in the year 1846, a most singular phenomenon was observed. The comet was at first seen as usual as a single body, but on its approach towards perihelion, it was, on the 13th of January, for the first time seen to be attended by a comet considerably fainter than

itself, at a distance of about 2′. This distance continued steadily to increase, while the difference of brightness of the two comets diminished, till the companion became as bright as the original. The latter then continued to increase in brightness, and exhibited a star-like nucleus, being decidedly superior in intensity of light to the original comet. This superiority, however, continued for a very short time; the original comet soon gained in brightness on its companion, which finally disappeared some time before the other ceased to be observed. Professor Plantamour, of Geneva, discussed rigorously the orbits described by each comet separately, and he found that during their companionship they kept very nearly the same distance from each other, and the line joining them moved parallel to itself.*

266. Other comets of short period recently discovered, we shall barely mention.

267. A comet discovered by M. Faye in 1843 describes an elliptical orbit in a period of 7½ years, and has been observed on its return in 1850.

268. A comet discovered by De Vico in 1844, and another by Brorsen in 1846, have each a time of revolution of about 5½ years.

269. Before closing this account of comets, we must make some mention of the remarkable one which appeared in the spring of 1843, which many of our readers may remember. Its nucleus or head was so near the horizon in the west at the time of its discovery, as soon as it grew dark, that it was observed very little in northern Europe, but its immense train was visible for several successive evenings spreading over an arc of nearly 60°. At Portland, in the United States, it was observed by Mr. Clarke, in the daytime, when scarcely more than 4° from the sun, and measures were made of the distance of its nucleus from the sun. Mr. Clarke describes it as "as well defined as the moon on a clear day, and resembling a perfectly pure white cloud without any variation, except a slight change near the head." An immense number of observations were made of this comet, many of which were made with the sextant on board ships that had a good view of it, and many interesting drawings of its appearance are preserved at the apartments of the Royal Astronomical Society. The sextant observations were also reduced under the auspices of the Society, and are printed in the Memoirs.

* See *Greenwich Observations for* 1845, for a series of observations of these companion comets.

270. It must be evident to all our readers, that the cometary branch of astronomy has been as well cultivated as every other, and among the most zealous and able of those who have devoted themselves to the discovery of these interesting bodies, to the computation of their orbits, and to the palæography of this branch of science, none holds a higher rank than our countryman Mr. Hind, so honourably distinguished for his planetary discoveries.*

CHAPTER VII.

THE FIXED STARS.

271. Having treated of the motions of the bodies of the solar system, viz. of those bodies that describe orbits referred to that body as a centre, we have now to speak of the stars, or of those comparatively fixed bodies that preserve sensibly from one age to another the same relative situation in the heavens, and are, therefore, popularly called fixed. In the sequel we shall find that this term is not strictly true, and that the greater number of the fixed stars have measurable motions of their own, but the designation is sufficiently accurate for ordinary purposes, and serves effectually to distinguish them from the planets, whose motions, as seen from the earth, are incomparably more rapid.

272. The stars are so distant from us that they are incapable of being distinguished except by their brightness, or to use the term generally applied, by their magnitudes. The separation of stars by magnitudes has been made from the earliest times altogether by estimation of their

* During the passing of these sheets through the press, the author has received a copy of the observations made under Mr. Bishop's direction at his Observatory in the Regent's Park, between the years 1839 and 1851. Mr. Bishop has had the good fortune, since the establishment of his observatory, to be assisted successively by Mr. Dawes and Mr. Hind, whose names are sufficiently well known to need no eulogium. The consequence is the publication of a volume forming one of the most valuable contributions to planetary, cometary, and sidereal astronomy, that have appeared for some time, and comprising probably more of original discovery than any astronomical volume whatever.

brightness with the eye, and this estimation being necessarily vague, the magnitudes as given in catalogues of stars are in some respects vague and indeterminate, and hardly accurate enough for the present purposes of sidereal astronomy.

273. A few stars of the heavens, preeminently bright, are called stars of the first magnitude, but this class includes very few, the number given in the Nautical Almanac list being only thirteen. Stars next to these in brightness, and differing only slightly, are said to be of the first-second or the second-first magnitude, accordingly as they by estimation appear to be nearer stars of the first or the second magnitude. The next class comprises stars of the second magnitude with its subdivisions, and so on to the third, fourth, &c., magnitudes. The lowest class visible to the naked eye consists of stars between the fifth and sixth magnitudes, though on very clear evenings good eyes may distinguish stars of the sixth magnitude. The classes of inferior magnitudes are estimated according to their relative brightness as seen in telescopes, and of course the estimations become still more vague on account of the different powers of the telescopes employed, and because a general view of them cannot be obtained for the purpose of direct comparison, as is the case with the naked eye when directed to the heavens.

274. We are indebted to Professor Argelander, Director of the Observatory at Bonn, for a complete classification of all the stars visible to the naked eye in the northern heavens according to their magnitudes, in a catalogue entitled *Uranometria Nova*, published a few years ago, which should be in the hands of every astronomical student.

275. Mr. Johnson, Director of the Radcliffe Observatory at Oxford, has also paid great attention to this subject, and has repeatedly estimated the magnitudes of all the stars observed by him, so as to obtain a much more definite scale than has been ever practicable before.*

276. For the better classification of the stars, the ancients divided them into fanciful groups, called constellations. The boundaries of these constellations were assigned from the supposed resemblance to the figures of men, animals, &c., and in many instances represent the deified heroes or heroines of antiquity, such as *Hercules*, *Perseus*, *Andromeda*, *Cassiopeia*. The most remarkable northern constellations

* On this subject see a valuable paper by Mr. Dawes in the *Notices of the Royal Astronomical Society*, vol. xi., p. 187.

called by the names of animals are *Ursa Major*, *Ursa Minor*, *Draco*, *Cygnus*, *Serpens*, *Aries*, &c., together with those crossing the zodiac, as *Aries*, *Taurus*, *Leo*, &c. The number of constellations thus named, which are included in Ptolemy's catalogue, is 48. Some stars of conspicuous brightness still retain the designations given to them by the Greek and Arabian astronomers, such as *Algenib*, *Achernar*, *Sirius*, *Rigel*, *Aldebaran*, *Capella*, *Arcturus*, *Antares*, *Spica Virginis*, *Regulus*, *Canopus*, *Fomalhaut*, &c. Bayer, an astronomer of the seventeenth century, the better to distinguish the stars in their respective constellations, assigned to them letters of the Greek, Italic, and Roman alphabets; the brightest star having affixed to it α, the first letter of the Greek alphabet, the second β, and so on, till, these being exhausted, recourse was had to Italic and Roman letters. Thus α *Aquilæ*, α *Lyræ*, are the brightest stars in the constellations *Aquila* and *Lyra*, and so on.

Notwithstanding the occasional confusion and inconvenience occasioned by these fanciful divisions of the stars into constellations, yet the general convenience is so great, and (what is of still greater importance) the stars have become by long usage so inseparably identified with their names, that it is impossible at the present time to make a complete revision of nomenclature. Yet, if any one is desirous of knowing what is thought on this subject by some of the most profound astronomers of the day, and the grounds on which a change of nomenclature has been thought desirable, he may consult a valuable paper by Sir John Herschel in the *Introduction to the Catalogue of Stars* published by the British Association.

277. We have already spoken of catalogues of stars, and it will be necessary to devote a few words to the explanation of their construction.

278. It has been explained that the places of objects in the heavens can be rid of all the effects of displacement to which the observations of them at the surface of the earth are liable, and can be referred to a fixed equinox and equator. It is usual in fixed observatories to reduce the observations of all stars to their mean positions for the beginning of the year of observation by the application of the corrections for precession, nutation, aberration, &c., and their places are then such as they would be if referred to the *mean position of the equinox and equator* for the beginning of the year; that is, the places of reference are thus fixed, and the

observed places of the stars should be theoretically the same for every observation. By collecting the separate observations of each star, therefore, a mean of the results will give with considerable accuracy the mean place for the beginning of the year. The mean right ascensions and polar distances of the stars are thus collected into a catalogue according to the order of their right ascensions, (that is, starting from 0^h according to the order of their diurnal revolution), and thus a catalogue of mean places of the stars observed during the year is found. It is usual also to add in separate columns the star's mean precessional motion in right ascension and north polar distance for one year, and, if it is known, the proper motion is also given. By these means it is easy to combine the results of several years' observations into one catalogue for some intermediate epoch, with no greater risk of error than is entailed by the mean effect of the trifling uncertainty of the actual motion of the equinox and of the proper motion in this interval, which for a few years is exceedingly small, in the present state of astronomy. Thus the results of all the star observations made at Greenwich, between the years 1836 and 1847, both inclusive, are embodied in two catalogues printed side by side, each containing the observations of six years. It was not thought prudent to combine them all into one catalogue on account of the objection arising from the possible uncertainties already spoken of, and the reader may hence judge of the jealous caution exercised in all matters subject to the smallest doubt. The proper motions of all such stars in the above catalogues as were observed by Bradley have been computed by the author of this treatise, and are published in the Memoirs of the Royal Astronomical Society. For all such stars, then, the places can be predicted with infallible accuracy for at least a century, certainly with smaller risk of error than belongs to a single observation.

279. Having thus explained the formation of a catalogue of stars, we will mention some of the most important collections, both ancient and modern. The first forms part of the famous *Almagest* of Ptolemy, supposed to have been compiled from the observations of Hipparchus made 267 years before, and reduced to Ptolemy's epoch by adding the amount of the precessional motion in longitude. This is the only ancient catalogue, and it contains 1022 stars.

280. The next in order is the catalogue of Ulugh Beigh, a Persian Prince, containing 1017 stars, compiled from his

observations made about the year 1437; and the next is the catalogue of the celebrated Tycho Brahe, containing 777 stars, arranged in 45 constellations, from observations made in the latter part of the sixteenth century.

281. The above catalogues have been re-edited and rendered more available for modern readers by the late Mr. Baily, and are contained in the thirteenth volume of the Memoirs of the Royal Astronomical Society.

282. Bayer's catalogue, containing 1762 stars, arranged in 72 constellations, we have before had occasion to notice. It was published in his *Uranometria* in 1603. Bayer introduced the classification of the stars by the letters of the alphabet.

283. The celebrated catalogue of Hevelius comes next, and contains 1888 stars derived from his own observations. The epoch is 1661. It was published in the *Prodromus Astronomiæ* in 1690, and is the last collection of star places derived from observations made without the help of the telescope. It is included in Mr. Baily's collection before mentioned.

284. Next follows the far more important catalogue of Flamsteed, which is the real basis of correct modern astronomy. (See our article on Observatories.) This catalogue was compiled from Flamsteed's observations made with the use of the telescope on a fixed mural arc, and was published by Flamsteed's executors in the *Historia Cœlestis Britannica* in 1725. It has also been re-edited by Mr. Baily.

285. The most celebrated catalogue of the eighteenth century is that of Bradley, containing nearly 3000 stars. It is contained in the *Fundamenta Astronomiæ* of the illustrious Bessel, published in 1818. This book is the greatest treasure possessed by astronomers, both from the goodness of the observations on which the catalogue is founded, and for the methods of reduction used by Bessel, which comprise all the refinements of modern mathematical skill.

286. Other fundamental catalogues are those of Piazzi and Groombridge. The former contains nearly 7000 stars, and the latter 4243, and the respective epochs are 1800 and 1810. Both these catalogues are of inestimable value.

287. The last original catalogues which we shall mention are those of Lalande and Lacaille, containing respectively 47,390 and 9766 stars for the years 1800 and 1750. Both these catalogues have been recently published by the British Association, the reductions of the observations

having been made under the authority and at the expense of that body.

288. The best modern *compiled* catalogue (that is, a compilation from all the best *original* catalogues before mentioned) is that of the British Association, containing 8377 stars, reduced with great accuracy, and containing the elements of reduction for obtaining the apparent place of a star for any given day. This catalogue replaced the *Astronomical Society's Catalogue*, which was published in 1827, and compiled by Mr. Baily. The appearance of this latter catalogue, with Mr. Baily's elaborate preface, of itself marked a new epoch in astronomy.

289. In connexion with the formation of catalogues of stars we are naturally led to treat of their *proper motions.* Imagine two catalogues of the same stars to be formed for two epochs, differing by a considerable interval; for example, the star catalogue included in Bessel's *Fundamenta*, which is deduced from Bradley's Observations, and whose epoch is 1755, and the Greenwich Catalogue of 2156 stars, whose mean epoch is about 1842. Now we can, by the theory of precession which has been previously explained, bring up the places of Bradley's stars to the epoch of the modern Greenwich Catalogue, without material error, and then, neglecting error of observation, which in the mean of several is very small, the results should be identical with the results of the latter catalogue. But we find this to be absolutely the case for very few stars, and, in a great many instances, the differences are so great that it is necessary to seek for some cause totally different from those apparent displacements of all the celestial bodies that we have thus far had occasion to consider. On the whole, also, the differences follow no law of sign or magnitude dependent on the position of the stars, by which we could represent their whole amount; and we are thus justified in considering them to be *real* or *proper motions* of the stars themselves. Indeed, it is exceedingly unlikely, from what we know of the motions of the bodies of the solar system, that any one of the stars is absolutely at rest, since on that supposition it could only remain so by the total absence of any attractive force from other bodies, that is, it must be at an infinite distance from every other. The observed differences then being divided by the number of years between the two epochs will give the annual proper motions of the stars. As a general rule it is found that the brightest stars, that is, the stars of the first and second

magnitudes, have the largest proper motions, but this rule has most remarkable exceptions. For instance, the two stars of 61 Cygni, whose distance has been discovered (see page 49), and whose magnitudes are not much above the sixth, have a common annual proper motion of about $5\frac{1}{3}''$, and the star No 1830 of Groombridge's catalogue has one still greater: μ and θ Cassiopeiæ have also large proper motions. All such stars, the reader will easily understand, are likely to be much nearer to us than those whose proper motions are insignificant, and are proper subjects for the attempt at the investigation of their parallaxes. The parallax of Groombridge 1830 appears, however, to be exceedingly small, and is scarcely recognisable by the refined observations that have been made of it by M. Peters of the Pulkova Observatory.

290. Amongst the most interesting of the phenomena ordinarily observable in the stars is that of the periodical variations of brilliancy belonging to some of them. In the time of Tycho, a star suddenly appeared in the constellation Cassiopeia, with a lustre exceeding that of stars of the first magnitude, and rivalling Jupiter and Venus when nearest to the earth. Its brightness decreased very rapidly, and at the end of sixteen months it was no longer visible. Its colour during this time underwent considerable variations. Another star of the same kind was observed in 1604, in Ophiuchus, and all the phenomena connected with it were similar to those of the former. At the present time, owing to the constant and systematic observations of the stars, a tolerably large list of *variable* stars has been collected, and the periods of maximum and minimum brightness of some of them have been ascertained. Of these β Persei is a remarkable specimen. Its whole period of change is rather less than $2^d\ 20^h\ 49^m$, during which time it varies in brightness from the second magnitude to the fourth. It continues at its state of greatest brightness for rather more than $2\frac{1}{3}$ days, and all its changes are confined to a few hours. The phenomena connected with this star strongly confirm the idea of a dark body revolving round it, and periodically obstructing a great portion of its light. Another variable star of the short period of $5^d\ 9^h$ nearly, is δ Cephei. β Lyræ has a period of either $6\frac{1}{2}$ days, or 13 days nearly, astronomers being rather divided in opinion. For a list of several other stars whose periods have been ascertained with tolerable certainty, the reader may consult Sir J. Herschel's Outlines of Astronomy, page 558, and we

would recommend the study of this interesting class of stars to any of our young readers who are provided with a telescope of moderate power, as a very delightful and profitable exercise. Mr. Hind, whose labours we have occasion so frequently to mention, has added four to the list. Such stars as those observed in the sixteenth century are more properly denominated *temporary* stars, the designation *variable* being confined to those that have a determined period of maximum and minimum brightness.

291. Of the different hypotheses devised for the purpose of accounting for the phenomena of variable stars, the two most probable are that either the stars, like the sun, rotate on an axis of revolution, and present in succession surfaces of greater and lesser illumination; or that they are attended by opaque bodies or planets revolving round them, and periodically obstructing some of their light. Bessel thought that he had discovered a variability in the proper motions of Sirius, Procyon, and one or two other stars which have been accurately observed for about a century, which could be explained by the disturbances incident to a planetary system, but the fact of the irregularity of proper motion does not appear to be quite confirmed.

292. The next class of stars presented to our notice are *double and multiple stars.* These may themselves be divided into two classes, viz. either those that are only *optically* double, that is, by being projected when seen from the earth on very nearly the same point of the heavens; or, those which are physically connected by gravity, and revolve round each other as the planets do round the sun. The researches connected with this portion of modern astronomy owe their origin to Sir W. Herschel, to whom we are indebted for the first catalogues of double stars. Since his time it has been most zealously prosecuted by his illustrious son and Sir James South, with several other astronomers in England, but especially by the great Russian astronomer Struve. Struve's large catalogue of double stars, in connexion with Sir John Herschel's observations at the Cape of Good Hope, has given us in this particular a complete survey of the northern and the southern heavens, and these eminent astronomers have left nothing for future observers but a scanty gleaning after their rich harvest.

293. The observation of a double star consists (in addition to notes of magnitude and colour) in the determination of the *distance* in seconds of arc of the two components, and of its *angle of position*, that is, of the angle which

the line of direction of its components makes with the meridian, or great circle joining the pole and the stars. By observations continued through the greater part of a century, it is certain that the angle of position of several double stars is not fixed, but that the line joining the components revolves constantly in the same direction, and some stars have actually completed a whole revolution since the commencement of accurate observation. Three of the most remarkable of these physically connected systems are, 61 Cygni and γ Virginis in the northern hemisphere, and α Centauri in the southern. The distance of the two stars of 61 Cygni has been tolerably constant since the earliest observations, and is about $15\frac{1}{2}''$; but the angle of position has varied by about 50° in seventy years. Now we have in our chapter on parallax (page 49) shown how the parallax, and therefore the distance of this star from the sun, has been measured by Bessel, and we need only repeat here that the mean value of the parallax (or angle subtended by the radius of the earth's orbit at the star, or rather the value of the angle whose tangent is $\frac{\text{rad. of earth's orbit}}{\text{dist. of star}}$), is $0{\cdot}348''$. Hence it follows, that the distance between the stars is greater than the radius of the earth's orbit in the proportion of $15\frac{1}{2}$ to $0{\cdot}348$, or of $44\frac{1}{2}$ to 1 nearly. The orbit, therefore, described by these stars round each other must of necessity be considerably greater than that of Neptune, the recently discovered exterior planet, whose distance from the sun is not much greater than 30 times the radius of the earth's orbit. Again, supposing the revolution of the stars round each other to be uniform, their time of describing the complete orbit will be $\frac{360}{50} \times 70$ years, or rather greater than 500 years. By means of these data, we find, by the application of Kepler's third law, the sum of their masses to be a little greater than one-third of the mass of the sun.

294. The distance of the two components of α Centauri was in 1834 about $17\frac{1}{2}''$, and decreases at the rate of about half a second per year, while the angle of position remains tolerably constant. Observations have not been continued long enough to determine the orbit described by the stars round each other with any accuracy; but if we assume with Sir J. Herschel, that the major axis must at the least exceed 24″, and with Henderson and Maclear (see page 51) that the parallax is rather less than 1″, then the real value of the axis of the orbit described is at least 13 times that of the earth's orbit.

295. γ Virginis is another remarkable double star whose components are physically connected and of nearly equal magnitudes. This star has had the elements of its orbit calculated by several astronomers with great exactness, according to the principles laid down by Sir John Herschel. Captain Smyth, so well known in the astronomical world, has devoted considerable time and labour to this investigation, both by methods of ordinary calculation, and by graphical methods, and it occupies a considerable space in his "Cycle of Celestial Objects."*

296. When the components of a double star are very unequal in magnitude, their colours are generally complementary to each other; thus, if the larger star be of a yellowish colour, the lesser one will appear bluish; but if the colour of the larger star incline to crimson, the other will be of a greenish hue; if again the smaller star be very much fainter than the other, the latter will not be affected by its light. Sir J. Herschel suggests the possibility of these colours being not purely the result of contrast, but of light differently tinted, emanating from the stars themselves: and if this should be the case, it is scarcely possible to imagine the glorious effects of coloured lights that would be produced during the revolutions of the planets attendant on such primaries. In many parts of the heavens, stars of a deep red colour occur, unattended by companions, but stars of a green or bluish tint have never been observed alone.

* We should fail in our duty, if we did not here mention that the discussion of the orbit of γ Virginis is also contained in a quarto work recently printed by Captain Smyth for private circulation entitled. "Ædes Hartwellianæ." This book, which is an admirable specimen of historical, antiquarian, and astronomical research, is as remarkable for the taste displayed in the embellishments contributed by the various members of the Captain's family, and supplies a want long felt by many, viz. a good account of Hartwell House, the residence of Dr. Lee. The munificence of the owner of this princely domain is well known, and through a long life he has omitted no opportunity of advancing not only the science of astronomy, but those of numismatics and antiquities, which are so well illustrated by his own admirably classed museum. It is a most pleasing feature in English science of every kind that many of the great discoveries of the age are due to the gratuitous zeal of individuals, and while we are justly proud of our National Observatory and its celebrated director, while we challenge all the observatories in the world to produce a mass of observations of equal value with those that have been made at Greenwich, yet we are equally proud to record the names of Herschel, Bishop, Lassell, and Lee, by whose gratuitous zeal and munificence the grander features of the astronomy of the day have been so wonderfully developed.

297. It was suspected by Sir W. Herschel that the sun has a proper motion in space, and this opinion, conformable as it is with analogy, has been proved to be true, though the illustrious Bessel, from the results of Bradley's Observations, thought otherwise. In any extensive catalogue of proper motions * in right ascension, if the very small and doubtful ones be rejected, it will be found that from about seventeen hours to five hours of R. A., the proper motions are on the whole such as to increase the right ascension, and that between five hours and seventeen hours, they tend to diminish them. Now this is just the effect that would be produced by the motion of the sun towards a point in the heavens of about seventeen hours R. A., for it is evident that the angular distances of all the stars in that part of the heavens towards which the sun is moving will be increased, while the angular distances in the opposite part of the heavens will be diminished. The stars will therefore be, with regard to right ascension, apparently thrown farther away from the seventeen-hour meridian, and the right ascensions greater than seventeen hours will be increased, while those less than seventeen hours will be diminished, which is conformable to the observed fact. Four elaborate investigations have been made of the position of the apex of solar motion; the first and second by Professor Argelander and M. Lundahl, the third recently by M. Otto Struve, and the fourth still more recently by the late Mr. Galloway. M. Otto Struve, by some very refined speculations, has deduced not only the *direction*, but a very near probable value of the *amount* of the solar motion. By his results it appears, that the sun is moving towards a point in the constellation Hercules, defined by R. A. 259°, and N. P. D. 55°, with an annual velocity of about 0·3″ measured in the arc of a great circle for a star at the unit of distance, situated 90° distant from the apex of motion.

298. More recently, M. Mädler of Dorpat has speculated on the determination of the point of the heavens where the central body is situated, round which the sun revolves. This point he places in the group of the Pleiades, a point lying considerably out of the plane of the milky way, a situation, as Sir J. Herschel remarks, in itself very improbable, "since it is almost inconceivable that any *general* circulation can take place out of the plane of the galactic circle."

* See for example the author's discussion of the proper motions of stars of the Greenwich Catalogue by comparison with Bradley's Observations. *Memoirs of Royal Astronomical Society*, vol. XIX.

299. Thus far we have treated of stars properly so called, that is, of celestial objects which, when viewed with telescopes of the highest power, present to the eye only single points of light. But there are scattered in various parts of the heavens other objects which never present the image of a star, and are seen very differently, according to the power of the telescope employed. Such objects are classed under the general term *nebulæ*. When viewed with telescopes of moderate power, such as those in use before the time of Sir William Herschel, they present generally the appearance of a small *nebulous* or *clouded* mass of light.

300. In the *Connaissance des Temps* (French Nautical Almanac) for 1784, Messier gave a catalogue of 103 nebulæ, and those found in his catalogue have been since generally referred to under the numbers assigned by him. In general they appeared to him simple nebulous masses, and received no farther subdivision. But the powers of Sir William Herschel's large reflecting telescopes showed a great variety of structure and form of these wonderful objects, and he was enabled to resolve all that he discovered in his general sweep of the northern heavens into the following classes:—

1st. Those decidedly resolved into clusters of separate stars. 2nd. Those which were not wholly resolved, but which apparently would be by the use of greater optical powers. 3rd. Those which in his telescopes showed no trace of resolution. 4th. Planetary nebulæ. 5th. Stellar nebulæ; and 6th. Nebulous stars.

We will devote a few words to each of these classes.

301. Of the first class a remarkable specimen is Messier 13, pictured in Sir J. Herschel's *Outlines of Astronomy*. It consists of a nearly spherical mass, containing a closely wedged multitude of stars compressed into a space not greater than 10′ in diameter, with a remarkable condensation of numbers and brightness towards the centre. Others of this class are of irregular figure, and generally contain fewer stars, and have less condensation towards the centre. They are mostly found either in or near the milky way.

302. Of the second class, (many of which, imperfectly resolved before, have yielded to the powers of Lord Rosse's gigantic 6 ft. reflector,) the shape is generally round or oval, the irregularities of their outlines being probably rendered invisible by the distance.

303. Of the nebulæ which are with great difficulty or not at all resolved in the most powerful telescopes, the most

remarkable are the *elliptic* and the *annular*. In the elliptic, the density always increases towards the centre, and the excentricities are of every magnitude, some being very flat or almost resembling a straight line, and others of very moderate ellipticity. The most remarkable of this class is that near ν Andromedæ (in the girdle) visible to the naked eye, and frequently mistaken for a comet.* It has been observed in America by Mr. G. P. Bond, assistant at the Observatory of Cambridge near Boston, with the great refractor recently added to that establishment, and some very remarkable peculiarities of its form, extent, and general structure, have been elicited by his description. (See *Trans. American Acad.* vol. iii.)

304. *Annular nebulæ* are very rare, but the most remarkable specimen is in Lyra, between the stars β and γ of that constellation. It consists of an elliptic ring of well-defined nebulous light, the axes being nearly as 5 to 4. The central vacant space contains traces of nebulous matter, and Lord Rosse with the 6 ft. reflector has detected a pretty bright star not far from the centre, and a few other minute stars. "In the annulus there are several minute stars, but there was still much nebulosity not seen as distinct stars."

305. *Planetary nebulæ* present circular or slightly oval disks, resembling planets, but with different degrees of definition at the borders. Very few of these objects have been discovered, and of these the greater number are in the southern hemisphere. One of the largest, however, is near the star β Ursæ Majoris, following it in the same parallel by about 12^m of R. A. Its apparent diameter is about $2\frac{2}{3}'$, which would imply a real diameter seven times greater than the orbit of Neptune, even supposing it no farther from us than 61 Cygni.

306. *Double nebulæ* sometimes occur; and if, as seems very probable, they are like physically connected double stars, that is, if they form two distinct systems of countless stars, each having its own centre of condensation, yet revolving round each other by the tie of gravitation, imagination quite fails to realise the vastness of the idea thus suggested to us.

307. *Nebulous stars*, as defined by Sir J. Herschel, consist of "a sharp and brilliant star, concentrically surrounded by a perfectly circular disk or atmosphere of faint light, in some

* Indeed I believe there are few astronomers who have not had the *misfortune* once at least in their lives to make this mistake. It is exactly like a comet without a tail.

cases dying away insensibly on all sides, in others almost suddenly terminated." 55 Andromedæ and 8 Canum Venaticorum are good specimens of this class. Lord Rosse gives a most interesting account of a star of the 8th magnitude of this class as seen with his 6 ft. reflector. "There is no trace of resolvability. The outer ring is seen on a pretty good night completely separated from the nucleus surrounding the brilliant point or star . . . There is a small dark space to the right of the star which indicates a perforation similar to that discovered in some others." Of ι Orionis, the characteristics are still more interesting.

308. Many of the nebulæ viewed by Lord Rosse are remarkable for a *spiral conformation;* that is, from a point of very great condensation nebulous streaks of variable density radiate in spiral convolutions, and in a way which denotes great regularity in the organisation of the structure. Nos. 51 and 99 of Messier, which are sketched by his lordship, are beautiful specimens. (See *Phil. Trans for* 1850, for drawings of the nebulæ as seen by Lord Rosse.)

309. We have now given a passing notice of all the classes of nebulæ which are visible in the northern hemisphere, but there are two remarkable phenomena visible with the naked eye in southern latitudes, called the *Magellanic Clouds,* which require mention. They are two cloudy masses of light, of a somewhat oval shape, but the larger deviates most from the circular form, and exhibits "the appearance of an axis of light, very ill defined, and by no means strongly distinguished from the general mass, which seems to open out at its extremities into somewhat oval sweeps, constituting the preceding and following portions of its circumference. . . . When examined through powerful telescopes, the constitution of the nubeculæ is found to be of astonishing complexity. The general ground of both consists of large tracts and patches of nebulosity in every stage of resolution, from light irresolvable with 18 inches of reflecting aperture, up to perfectly separated stars like the milky way, and clustering groups sufficiently insulated and condensed to come under the designation of irregular, and in some cases pretty rich clusters. But besides these, there are also nebulæ in abundance, both regular and irregular; globular clusters in every state of condensation; and objects of a nebulous character quite peculiar, and which have no analogy in any other part of the heavens." *

* *Outlines of Astronomy,* p. 613.

310. We have now completed the plan which was proposed at the outset of this work. Beginning with those remarkable features and phenomena of the heavens which are forced on the attention of every one alike, we have endeavoured to trace the successive steps in the reasoning by which the diurnal and apparent are separated from the real motions of the heavenly bodies. We have then given, as fully as our space permits, an account of the operations by which the figure and dimensions of our own globe are ascertained, and by which it becomes the basis for the measurement of the magnitudes and distances of the other planets. An account has then been given of those instruments principally employed for determining relatively and absolutely the positions of the heavenly bodies, and of the successive corrections which it is necessary to apply to the observed places before they can be rendered available for the use of the theoretical astronomer. A specific account is then given of the bodies of the solar system, viz. of the great centre of attraction, the sun, and of all those bodies that revolve round him in elliptical orbits, and in this part of the work we have endeavoured to introduce the reader to some popular notions respecting physical astronomy, or the effects of the universal law of gravitation in producing not only the motions which rough observations of the planets exhibit to us, but those minute deviations which only refined observations can detect and the most refined and complicated analysis can extract as consequences of the general law. Lastly, the student has been introduced to a popular view of sidereal astronomy, and to the wonders which such telescopes as Lord Rosse's have revealed to us.

If an intelligible idea has been gained of the processes and results of astronomical science thus briefly sketched,—if the student has learnt at all to appreciate the labours of those learned men who have with unwearied industry, each in his own department, helped to build up the noble structure of the theory of the heavens, such as it is exhibited to the more advanced student at present; but above all, if he has learnt to adore more profoundly the infinite wisdom of the Almighty architect who, by his word, created all these wonders that we are lost in contemplating, this little book will not have been written in vain; and the author will not regret having added one volume more to the list of those handbooks of popular astronomy that seem even now too numerous. The study of Astronomy, to benefit the reasoning powers of the student, must show the nature of the



processes by which the grander features of the science, which all delight to contemplate, have been arrived at; and must at the same time point out with some accuracy the boundaries of our knowledge. We deal with a science which abounds in the marvellous and illimitable. We gird the earth with a measuring line of indisputable accuracy, and we measure the distances and determine the weights of the planets with wonderful precision, and we finally enter within the regions of what we might have supposed infinite space, and find the distances and determine the masses of the stars. But these successive steps have been gained by no empirical processes, but by the sure yet cautious application of inductive principles. As the *quantity* on which the phenomenon we are seeking depends becomes less and less, our jealous scrutiny of all the sources of error and delusion in our instruments and means of observation becomes greater, and the analytical processes by which we extract it from our observations become more refined.

We have endeavoured to familiarise the mind of the student with such principles in the *first* place, and in the *second* to give a sufficient number of examples of the most interesting facts and phenomena of the science If the *principles* be understood, a multitude of books will be found which will give the *results* in a more taking and interesting form. Our aim has been not to write a merely popular, but a useful book, which may serve as an introduction, both to such books as Sir J Herschel's Outlines of Astronomy, which embraces the whole subject in a popular shape for one class of readers, and for another class to books which treat of the mathematical and systematic treatment of the subject.

THE END.

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